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Attracting profit shifting or fostering innovation? On patent boxes and R&D subsidies*

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Abstract

Many countries have introduced patent box regimes in recent years, offering a reduced tax rate to businesses for their IP-related income. Patent boxes are supposed to increase innovative activity, but they are also suspected to aim at attracting inward profit shifting from multinational firms. In this paper, we analyze the effects of patent box regimes when countries can simultaneously use patent boxes and R&D subsidies to promote innovation. We show that when countries set their tax policies non-cooperatively, innovation is fostered, at the margin, only by the R&D subsidy. The patent box tax rate is instead targeted at attracting international profit shifting, and it is optimally set below the corporate tax rate. With cooperative tax setting, the optimal royalty tax rate is instead equal to, or even above, the statutory corporation tax. Hence, patent box regimes emerge in the decentralized policy equilibrium, but never under policy coordination. We also show that enforcing a nexus principle, as proposed by the OECD, is helpful to mitigate harmful tax competition. However, the best policy would be to eliminate patent boxes altogether.

Keywords: corporate taxation; profit shifting; patent boxes; R&D tax credits; tax competition

JEL classification: H25, H87, F23

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1 Introduction

The importance of innovation for economic development and growth has received major attention in the last 20 years, and providing public support for the underlying R&D investment has become an important policy objective. Out of 36 OECD countries, 83% offered R&D tax incentives in 2018, up from 53% in the year 2000 (Appelt et al., 2019, Figure 2). In the same period, the average implicit R&D subsidy rate for profit-making firms has increased from 4-5% in the year 2000 to 14-16% in 2018.[1] The economic reason for rising R&D subsidies are their impact on innovation, economic growth and market efficiency (see Bloom et al., 2019, for an overview). Moreover, increasing market integration strengthens both international R&D spillovers, and the case for R&D subsidies.[2]

At the same time, profit shifting by multinational corporations (henceforth MNCs) remains high on the agenda of both academics and policymakers. A large empirical literature has established convincing evidence that MNCs use various channels to transfer their profits from high-tax to low-tax countries.[3] Current attempts to quantify the overall extent of profit shifting estimate that close to 40% of all profits of MNCs are shifted to tax havens (Tørsløv et al., 2018). With the OECD's (2013) action plan against base erosion and profit shifting (BEPS), the topic has also taken center stage in policy debates.

A policy instrument that connects both policy issues are so-called 'patent boxes', which introduce reduced corporate tax rates for income derived from patents and other intellectual property. These have become a hotly debated issue after many countries, particularly in Europe, but also elsewhere, opted for them.[4] Table 1 shows that 15 European countries have introduced such patent boxes by the year 2020, with the most recent patent box regimes introduced in Poland and Switzerland. In most countries, patent boxes were introduced with the stated aim of fostering domestic R&D. However, empirical research has established that placing patents in low-tax countries is one prominent channel by which MNCs engage in profit shifting (Karkinsky and Riedel, 2012; Baumann et al., 2020). Therefore, it is widely believed that attracting inward profit shifting from MNCs is a further main reason for introducing patent box regimes.

Given these twofold effects of patent box regimes, the policy debate surrounding them has been highly controversial. The current policy compromise (OECD, 2015) is to permit patent tax regimes, but to confine the preferential tax treatment to income from patents

[1]See Appelt et al. (2019, Figure 7). This figure uses the OECD R&D tax incentives database to aggregate various policies to promote R&D to an implied R&D subsidy rate.

[2]See Haaland and Kind (2008) for a theoretical analysis of the effects of market integration on R&D subsidy choices, and Fracasso and Marzetti (2015) for empirical evidence that higher trade flows lead to disproportionate increases in international R&D spillovers.

[3]See Cristea and Nguyen (2016) and Davies et al. (2018) for recent examples of this literature, and for further references.

[4]In the United States, the 2017 tax reform includes a concessionary tax on foreign-derived intangible income (FDII). Though not officially labelled so, this provision operates like a patent box with a maximum tax rate of 13% (Mintz, 2018).

Table 1: Patent box regimes in Europe (2020)

country	year of introduction	qualifying assets			patent box tax rate (%)	statutory CIT rate (%)
		patents	software	other		
Belgium	2007	x	x		4.5	29.6
Cyprus	2012	x	x	x	2.5	12.5
France	2000	x	x		10	34.4
Hungary	2003	x	x		0, 4.5[a]	9
Ireland	1973/2015	x	x	x	6.25	12.5
Italy	2015				14	28
Lithuania	2018	x	x		5	15
Luxembourg	2008	x	x		5.2	26
Netherlands	2007	x	x	x	7	20
Poland	2019	x	x		5	19
Portugal	2014	x			10.5	21
Slovakia	2018		x		10.5	21
Spain	2008	x	x		10	25
Switzerland	2020	x			by canton[b]	by canton[b]
United Kingdom	2013	x			10	19

[a]: 0% for capital gains and 4.5% for royalty income
[b]: patent box rates and regular CIT rates differ by canton; patent box reductions are up to 90% of regular cantonal CIT rates

Sources: OECD Dataset Intellectual Property Regimes (2018), Tax Foundation Report: Patent Box Regimes in Europe (2019). Years of patent box introduction from Fabris (2019, 40-41).

that have been developed to a substantial degree in the country granting the tax rebate *(nexus requirement)*. This policy compromise follows empirical findings that the strategic (re-) location of patents within MNCs is most sensitive to patent box regimes that do not have a nexus requirement (Alstadsæter et al., 2018). However, even if tax concessions are confined to IP-related income that meets the nexus requirement, a harmful *race-to-the bottom* for the taxation of IP-related income that results in revenue losses for all countries involved is still a possible, and perhaps even a likely, outcome (Griffith et al., 2014).

In this paper, we analyse tax policy towards R&D incorporating the simultaneous goals of governments to foster innovation and to attract tax base. In particular, we argue that the analysis of patent box regimes must account for the fact that countries already have a set of policy instruments to promote innovation, such as R&D subsidies or tax credits, which they are increasingly using (see Appelt et al., 2019, referenced above). We show that if patent box regimes and R&D tax subsidies are introduced as separate policy instruments in an optimal tax framework, then a clear targeting result emerges: Direct R&D subsidies will be the marginal instrument to increase R&D, whereas patent box regimes are used to attract MNCs' profit shifting. In sum, patent box regimes emerge from our analysis as a classical beggar-thy-neighbor policy, and this is revealed by incorporating countries' simultaneous choice of R&D subsidies. Accordingly, our analysis provides a theoretical underpinning for the claim that existing patent box regimes should be eliminated, and replaced by well-designed R&D tax credits (e.g., Bloom et al., 2019, p. 171).

To formally derive our results, we set up a multi-country model of tax competition with multi-affiliate MNCs where the parent company of an MNC actively develops a patent that can be used in all its affiliates. Therefore, the MNC parent receives IP-related income from each of its subsidiaries. Each government has three tax instruments at its disposal: the standard corporate tax rate, the R&D subsidy rate, and a special tax rate that applies to the royalty (or patent) income of the parent firm. In the simplest case where R&D production generates no pure profits, we show that when countries unilaterally choose their optimal policies, each country will set the tax rate on royalty income below the statutory corporate tax rate, in order to attract inward profit-shifting. Consequently, a patent box regime emerges endogenously under tax competition.

We compare these results with the set of policies that emerges when countries coordinate all policy instruments to maximize global welfare. When there are no pure profits in the R&D sector, the optimal coordinated tax policy features an R&D subsidy, and it equates the tax rate on patent income to the standard corporate tax rate. With pure profits from R&D, the coordinated royalty tax rate is even higher than the statutory tax rate to tax the rents generated in the R&D sector. Hence, patent box regimes are never part of a coordinated set of tax policies.

In an extension of our benchmark model, we allow firms' R&D units and patents to move across jurisdictions. In particular, we compare the setting of non-cooperative policies in the case where patents are bound in the country where R&D has occurred, but R&D units are internationally mobile, to the case where patents can be relocated across countries, but R&D units stay in the country of the multinational's headquarters. We argue that the first case is relevant for patent box regimes with a nexus requirement, whereas the second case applies to patent boxes without this requirement. We find that patent box regimes are likely to be more harmful in the second case where patents, rather than R&D units, are internationally mobile. This result implies that the nexus requirement is indeed a suitable coordination measure, which dampens the aggressiveness of patent box regimes. At the same time, however, policy competition with a nexus requirement leads countries to engage in an inefficient *race to the top* in the setting of R&D subsidies. This suggests that the observed recent increase in the use of R&D tax credits may at least partly be driven by the strategic motive of governments to attract the internationally mobile R&D units of large corporations.

Our paper contributes to the literatures on patent box regimes on the one hand, and on R&D subsidies on the other, which have so far been almost completely separated. To the best of our knowledge, there is no theoretical analysis that endogenously derives patent box regimes in an optimal tax framework.[5]

[5]Sharma et al. (2020) analyze the location of patents when the R&D process is uncertain. Their

The empirical literature on patent boxes has come to different conclusions as to how effective patent boxes have been in terms of both fostering R&D and attracting profit shifting. Bradley et al. (2015) find some positive effect of patent box regimes on patenting activity, but no effect on profit shifting. In contrast, Gaessler et al. (2019) find no effect of patent boxes on innovation itself, but a positive effect on trade in patents. Similarly, Bornemann et al. (2020) find for the Belgian IP box that innovative activity increases, but patent quality decreases and all real effects are driven by domestic firms. In contrast, MNCs largely enjoy reductions in their effective tax rates. Koethenbuerger et al. (2018) decompose the total effect of introducing patent box regimes on national tax bases using a double difference-in-difference analysis. They attribute most of the increase in the tax base to inward profit shifting, and only a smaller fraction to the induced increase in R&D.

Alstadsæter et al. (2018) show that the specific design of patent box regimes in individual countries has important implications for how strongly patenting activity on the one hand, and profit-shifting on the other, respond to the tax incentives offered by the patent box. Chen et al. (2019) look at real activity and find that those high-tax countries that introduce the most aggressive patent boxes experience less income shifting out of the country, more investment, and increased employment. Schwab and Todtenhaupt (2019) empirically examine the cross-border effects of patent boxes on real R&D activity in neighboring countries and show that these effects depend critically on whether patent boxes are designed with or without a nexus requirement. Finally, Davies et al. (2020) estimate that patent box regimes increase the average success rate of patent applications, but the effect turns negative for frequent innovators.

A separate literature studies the effectiveness of R&D incentives by means of tax credits or direct subsidies. Bloom et al. (2019) provide a recent review of this literature and evaluate its policy implications. Several recent studies have used administrative data and quasi-experimental designs to evaluate the effectiveness of national R&D measures (Rao, 2016; Guceri and Liu, 2018).[6] Boesenberg and Egger (2017) are one of the few empirical studies of R&D incentives that also incorporate patent box regimes. Controlling for a broad range of simultaneous R&D incentives, they find that patent box regimes have a very low, and sometimes even negative effect on the filing of patents, and thus, on innovation activity. Finally, Knoll et al. (2019) study cross-border effects of input-related R&D tax incentives and show that MNCs redirect the R&D investment of their affiliates to subsidizing countries, with little change in the total R&D activity of the entire group. These empirical results point to the importance of simultaneously incorporating all policies to promote R&D, and to the relevance of international policy competition for

analysis is positive, however, and focuses on the effect of asymmetric loss-offset rules on patent location. Patent box regimes are only modelled implicitly, and optimal policies are not endogenized.

[6] As stressed by Chen et al. (2021), an unbiased estimate of the effectiveness of R&D tax incentives must account for the possibility that firms merely relabel expenses as R&D. For a Chinese tax reform to promote R&D, they estimate that 30% of the increase in reported R&D was due to such relabelling.

mobile R&D units. Both features are incorporated in our analysis.

We also contribute to the theoretical literature on optimal corporate tax policy with several tax instruments, and in the presence of both capital mobility and profit shifting. Examples of this literature are Peralta et al. (2006), Hong and Smart (2010), Haufler et al. (2018), Choi et al. (2020), and Mongrain and van Ypersele (2020). The novel element in this paper is to add a R&D decision of the multinational firm, as well as policies to promote this activity. Hence, we differentiate not only between policies to attract real capital versus profit shifting, but also between policies that attract capital to either the final-good or the R&D sector.

We proceed as follows. Section 2 presents our model of the MNCs' R&D, production and profit-shifting decisions. Section 3 analyzes the optimal set of tax policies when countries choose their policies non-cooperatively. Section 4 contrasts these results to the case where countries coordinate their tax policies. Section 5 extends the analysis by comparing policy competition when either R&D units or patents are mobile internationally. Section 6 discusses further aspects of the choice between patent box regimes and R&D policies. Section 7 concludes.

2 The model

2.1 The basic framework

We assume an international economy with n large countries. All countries are identical and each is equipped with a fixed stock of internationally mobile capital $\bar{k}$. Each country is home to an MNC and hosts both the headquarters (henceforth HQ) of this MNC and its R&D unit. The R&D unit conducts research to improve the MNC's production technology. Each MNC maintains one production affiliate in each of the n countries, which produces a homogenous consumption good. In each country, the affiliates of all MNCs sell the final good y to local customers at a price that is normalized to unity. Finally, each country i is home to one national firm that earns profits π_N^i. These profits are exogenous from the perspective of the national firm, but they are affected by spillovers from the MNC's R&D activity.

Capital is internationally mobile, but the location decisions of all MNC entities are exogenously given. In particular, the R&D unit of a MNC is always located in the HQ country in our benchmark model.[7] Each MNC uses capital k in its R&D process, and for producing the final output good. The rental cost of capital is determined by the endogenous world market interest rate r.

The R&D unit of the MNC headquartered in country i produces an MNC-specific public good q^i that enhances, as technological quality, output production in all affiliates

[7] In Section 5, we will relax this assumption and make the R&D unit mobile internationally.

of the MNC.[8] The capital input k_R into R&D has a positive and non-increasing marginal productivity, leading to a production function for innovation quality q^i of

$$q^i = q^i(k_R^i) \quad \text{with} \quad q_k^i > 0, \quad q_{kk}^i \leq 0. \tag{1}$$

The capital input for the production of the final output good is denoted k_j^i, where the subscript j stands for the host country of the affiliate, and the superscript i stands for the HQ country of the MNC. Given the technological quality q^i from the R&D process, affiliate j of MNC i generates output y_j^i according to the production function

$$y_j^i = f(k_j^i, q^i) \quad \text{with} \quad f_k > 0,\ f_{kk} < 0,\ f_q > 0,\ f_{qq} < 0,\ f_{kq} > 0. \tag{2}$$

Hence, capital and technological quality both have positive, but decreasing marginal productivities. Furthermore, technological quality and capital inputs are complements, i.e., the marginal productivity of capital k_j^i increases with technological quality q^i.

R&D investment in country i provides a positive externality on the domestic economy. For our purposes, it is immaterial whether the positive externality is driven by a direct technology spillover or, for example, by a better trained workforce that benefits from learning-on-the-job spillovers. Therefore, we simply capture the positive externality by assuming that the profits of the national firm π_N^i depend positively on domestic R&D investment k_R^i, that is $\pi_N^i(k_R^i)$ with $\partial \pi_N^i / \partial k_R^i > 0$.[9]

In exchange for using the technological quality produced in the R&D unit, each production affiliate pays royalties to the R&D unit. The tax authority in the deducting country, i.e., the host country j of a production affiliate, observes the average quality of technology and its marginal product for all MNE affiliates that operate in the local market. Tax authorities then base their arm's length transfer price p_j on this average marginal productivity of R&D:[10]

$$p_j = \frac{f_q(k_j^j, q^j) + (n-1) f_q(k_j^i, q^i)}{n} = f_q, \tag{3}$$

where the last equality follows from symmetry. The average R&D transfer price p_j (the 'blueprint price') is exogenous from the perspective of each MNC.

[8] Hence, our model assumes that the R&D process leads to deterministic outcomes. We discuss the implications of introducing uncertainty about the returns from R&D in Section 6.

[9] The empirical literature provides strong evidence for such positive externalities on the home economy; see, e.g., Bloom et al. (2013). Moreover, these externalities can be derived from a general estimation strategy without imposing specific spillover channels (Eberhardt et al., 2013).

[10] While imposing a 'correct' arm's length price is difficult in practice, assuming that tax authorities can observe (or estimate) the average productivity of a R&D investment in their country seems to be a good first approximation. If tax authorities can only infer a lower bound for the productivity increase, more productive MNCs can shift some profits without incurring tax planning costs (Bauer and Langenmayr, 2013). This additional source of profit shifting should not change our qualitative results, however.

Each MNC can, however, deviate from the average royalty payment $p_j q^i$ in a host country j by (falsely) claiming that its technological quality differs from the average quality. The deviation from the arm's length transfer price is labelled a_j^i, with $a_j^i \gtrless 0$. The MNC-specific royalty payment is then $p_j q^i + a_j^i$, thus permitting the MNC to transfer an amount of profits a_j^i from the production affiliate in country j to its R&D unit in country i. This activity, however, involves tax planning costs C, which can be interpreted as documentation or negotiation costs. Following the literature, we assume that the tax planning costs C are a convex and, for simplicity, quadratic function of the amount a_j^i by which royalty payments are misdeclared:[11]

$$C = C(a_j^i) = \frac{\beta}{2}(a_j^i)^2. \tag{4}$$

The cost parameter β measures how difficult it is to justify deviations from the arm's-length payment and shift profits. It can therefore be interpreted as an inverse measure for the degree of goods and capital market integration in the world economy. The more integrated the MNC is (the lower is β), the easier it is to hide excessive royalty payments (i.e., profit shifting) in its global, intra-firm trade flows.[12]

The government of each country i has three tax instruments at its disposal. First, it taxes the profits of all production affiliates that are active in country i, using the statutory corporate tax rate t_i. Second, each government can set a special royalty tax rate τ_i that falls on the returns of the R&D unit residing in country i. If the royalty tax rate is lower than the corporate tax rate, $\tau_i < t_i$, we obtain a preferential treatment of royalty income that corresponds to a patent box (see Table 1 in the introduction).[13] Finally, the government can grant a direct R&D subsidy s_i, or equivalently a R&D tax credit, that reduces the rental costs per unit of capital in the R&D sector.[14]

For simplicity, we assume that all capital investment is financed by equity. Following most OECD countries' tax codes, the cost of equity is not deductible from the corporate tax base. We also adopt the ruling international standard by which the residence country of an MNC exempts foreign incomes from tax. Hence the profits of an affiliate are taxed in the affiliate's host country, and dividend payments to the HQ country (i.e.,

[11]In general, results in the profit shifting literature are sensitive to the exact specification of tax planning costs. However, Juranek et al. (2018) have shown that the analysis of transfer pricing in intangibles, such as royalties, is largely insensitive to the modelling of these costs. In addition, these authors show that MNCs effectively engage in tax planning without manipulating any real investment decision. This is in line with our approach of assuming a lump-sum shifting of royalty incomes.

[12]A good example of market integration is the EU Interest and Royalties Directive from 2003. It facilitated capital flows between member states of the European Economic Area (EEA) by abolishing withholding taxes on interest and royalty payments between EEA countries.

[13]For brevity, we use the term 'royalty tax rate' as being synonymous with 'patent box tax rate'.

[14]R&D tax credits lead to a dollar-for-dollar reduction in the firm's tax payments. Therefore, direct R&D subsidies and R&D tax credits have analogous effects as long as the net tax payment of the MNC in its HQ country remains positive. The symmetry of the model ensures that this condition is fulfilled throughout our analysis.

profit repatriations) do not cause additional tax payments in the country of the parent company.[15]

2.2 The multinationals' choices

The R&D unit of an MNC in country i earns revenues from invoicing royalty payments in all production affiliates of the MNC. These royalty payments consist of the 'true' transfer price p_j applied to the MNC's technological quality q^i, plus the profit-shifting component a_j^i. The tax rate on the profits of the R&D unit is the royalty tax rate τ_i. The capital costs of the R&D unit are given by the world market interest rate r, less the R&D subsidy s_i per unit of capital investment k_R^i. The after-tax profits of the R&D unit in country i, resulting from royalty payments by the producing affiliate in country i and in $(n-1)$ symmetric affiliate countries j are thus given by

$$\pi_R^i = (1-\tau_i)\left\{[p_i + (n-1)p_j]q^i + a_i^i + (n-1)a_j^i\right\} - (r - s_i)k_R^i. \tag{5}$$

A production affiliate in country j earns after-tax revenues from selling output y_j^i, and it deducts royalty payments at the host country's corporate tax rate t_j. In addition, the affiliate carries non-deductible capital costs rk_j^i and the tax planning costs $C(a_j^i)$. For simplicity, and without affecting any of our qualitative results, we assume that tax planning costs are not tax-deductible.[16] The after-tax profits of a production affiliate of MNC i in country j are then

$$\pi_j^i = (1-t_j)[f(k_j^i, q^i) - p_j q^i - a_j^i] - rk_j^i - \frac{\beta}{2}(a_j^i)^2. \tag{6}$$

Total after-tax profits of an MNC headquartered in country i, Π_M^i, equal the sum of all affiliates' profits in (5) and (6). The MNC maximizes Π_M^i by choosing investment in the R&D unit (k_R^i), investment in all producing affiliates (k_j^i), and the levels of income shifting to the R&D unit (a_j^i). Using symmetry gives

$$\begin{aligned}
\max_{k_R^i, k_i^i, k_j^i, a_i^i, a_j^i} \quad \Pi_M^i \quad &= \pi_R^i + \pi_i^i + (n-1)\pi_j^i \qquad (7)\\
&= (1-t_i)f(k_i^i, q^i) + (n-1)(1-t_j)f(k_j^i, q^i) + (t_i - \tau_i)(p_i q^i + a_i^i) - \frac{\beta}{2}(a_i^i)^2\\
&+ (n-1)[(t_j - \tau_i)(p_j q^i + a_j^i) - \frac{\beta}{2}(a_j^i)^2] - r[k_R^i + k_i^i + (n-1)k_j^i] + s_i k_R^i.
\end{aligned}$$

[15] The exemption method has traditionally been applied in countries of continental Europe. During the last decade, the United Kingdom, Japan and the United States (in its 2017 tax reform) also switched to the exemption method. Therefore, with few exceptions (Chile, Israel, Mexico, and South Korea), the exemption method is now the dominant scheme of taxing MNCs in OCED countries.

[16] In reality, tax planning costs comprise deductible items such as the costs of tax consulting and legal advice, but also non-deductible components such as expected penalties in case of detected misconduct.

Capital investment k_j^i in affiliate j follows from the first-order condition

$$\frac{\partial \Pi_M^i}{\partial k_j^i} = (1 - t_j) f_k - r = 0. \tag{8a}$$

Each producing affiliate balances marginal after-tax returns against marginal capital costs. As revenues are taxed, whereas the costs of equity are non-deductible, the statutory tax rate t_j distorts each affiliate's production decision and reduces investment. This subjects countries to tax competition via the statutory tax rate t_j.

The first-order condition for the optimal R&D investment of an MNC in country i can be simplified using the definition of the arm's-length price, which implies $p_i = f_q$ from (3) and $p_i = p_j$ from symmetry. This yields

$$\frac{\partial \Pi_M^i}{\partial k_R^i} = (1 - \tau_i) n f_q q_k^i - (r - s_i) = 0. \tag{8b}$$

In our setting, R&D investment is an MNC-specific public good. Therefore the first-order condition (8b) corresponds to a Samuelson condition stating that the aggregated marginal net returns from using the developed technology in the final-good production of all affiliates must equal the marginal net investment costs. In the same way as in (8a) above, the royalty tax rate τ_i reduces R&D investment, as revenues are taxed but capital inputs are not tax deductible. However, the R&D subsidy s_i counteracts this distortion.

Finally, the MNC determines its level of profit shifting via royalty payments from the affiliate in j to the R&D unit in i. There is no public-good character in profit shifting, and on the margin, there are no economies of scale in the tax planning costs.[17] The deviation from the arm's-length price is country-specific and needs to be justified and defended in each affiliate against the local tax authority. Hence, we obtain the standard transfer pricing formula

$$a_j^i = \frac{t_j - \tau_i}{\beta}. \tag{8c}$$

A positive tax rate differential $t_j - \tau_i$ between a production affiliate j and the R&D unit sets incentives to shift profits, and thus causes an excessive royalty payment a_j^i. Higher costs for tax planning, as measured by the parameter β, reduce the net gains from profit shifting and hence reduce a_j^i.

[17]This does not preclude that tax planning causes some fixed costs in the HQ country besides the variable costs in each affiliate. Such fixed costs would not matter for our analysis, as long as the MNC is sufficiently large so that the tax savings from profit shifting overcompensate the fixed costs.

2.3 Capital market equilibrium

The world market interest rate r is endogenously determined in our symmetric n country model. World capital market clearing requires

$$n[k_i^i + (n-1)k_i^j + k_R^i] = n\bar{k}. \tag{9}$$

Together with the capital demand functions characterized by the first-order conditions (8a) and (8b), this condition determines the interest rate r. Appendix A.1 derives the effects on the market interest rate for each of our policy instruments. These are:

$$\frac{dr}{dt_i} < 0, \quad \frac{dr}{d\tau_i} < 0, \quad \text{and} \quad \frac{dr}{ds_i} > 0. \tag{10a}$$

The induced capital outflow from an increase in the corporate tax rate of country i increases capital supply in the other countries and triggers a decrease in the world market interest rate to accommodate the FDI flow. The same applies for an increase in the tax rate on royalty payments τ_i in country i. In contrast, an increase in the R&D subsidy s_i raises global capital demand and leads to a higher interest rate.

For investments into R&D, we get (see Appendix A.1):

$$\frac{dk_R^i}{dt_i} > 0, \quad \frac{dk_R^i}{ds_i} > 0, \quad \text{and} \quad \frac{dk_R^i}{d\tau_i} = -nf_q q_k \frac{dk_R^i}{ds_i} < 0. \tag{10b}$$

An interesting result of our setup is that a higher statutory tax rate t_i increases R&D investment. A higher tax t_i reduces investment in the production affiliate i and thus reduces the world market interest rate from (10a). This lowers the costs of R&D investment, whereas the returns to R&D investment (i.e., royalty incomes) are unaffected by the change in t_i.[18] Similarly, higher R&D subsidies reduce the costs of investing into R&D, and raise k_R^i. In contrast, an increase in the tax rate for royalty income, τ_i, lowers R&D investment. Note, finally, that the last two effects are linearly dependent on each other.

Capital investment in production affiliates reacts according to

$$\frac{dk_i^j}{dt_i} < 0, \quad \frac{dk_i^j}{ds_i} > 0, \quad \text{and} \quad \frac{dk_i^j}{d\tau_i} = -nf_q q_k \frac{dk_j^i}{ds_i} < 0, \tag{10c}$$

where we assume that the number of countries n is sufficiently large.[19]

A higher tax rate t_i reduces after-tax sales in the production affiliate i, discouraging capital investment k_j^i. In contrast, a higher R&D subsidy fosters technological quality and renders capital investment k_i^j more productive. Therefore, investment k_i^j is increasing in s_i.

[18] In the absence of a patent box, i.e., with $t_i = \tau_i$, the returns to R&D *are* affected by t_i, and R&D falls in response to a higher statutory tax rate.

[19] This ensures that the direct effects of policy instruments dominate the offsetting impact from the change in the interest rate. See Appendix A.1.

The opposite holds for the tax rate τ_i on royalty payments, which discourages investment in R&D and makes capital investment k_i^j less profitable. Once again, the effects of the R&D subsidy s_i and the tax rate on royalty income τ_i are linearly dependent.

Finally, there are two sets of externalities on the other countries. The first set is given by the effects on R&D investment in countries $j \neq i$:

$$\frac{dk_R^j}{dt_i} > 0, \quad \frac{dk_R^j}{ds_i} < 0, \quad \text{and} \quad \frac{dk_R^j}{d\tau_i} = -n f_q q_k \frac{dk_R^j}{ds_i} > 0. \tag{10d}$$

An increase in the corporate tax rate of country i boosts R&D investment of all MNCs, k_R^j, no matter where they are headquartered. This is because the higher corporate tax rate in country i reduces the world market interest rate and thus fosters profitability of R&D investments worldwide. In contrast, an increase in the R&D subsidy in country i increases the world market interest rate and hence makes R&D investment in all other countries $j \neq i$ less profitable. This is the standard capital-importing externality of a capital subsidy. The reverse applies to an increase in the royalty tax rate τ_i.

The second set of externalities falls on the capital investment in producing affiliates:

$$\frac{dk_j^m}{dt_i} > 0, \quad \frac{dk_j^m}{ds_i} < 0, \quad \text{and} \quad \frac{dk_j^m}{d\tau_i} = -n f_q q_k \frac{dk_j^m}{ds_i} > 0. \tag{10e}$$

Higher tax rates t_i and τ_i reduce the world market interest rate and increase capital investment k_j^m in all affiliates $j \neq i$ and for all MNCs m. The opposite holds for an increase in R&D subsidies s_i.

3 Optimal policies under tax competition

We are now ready to analyze optimal tax policies. We start with the case where each government non-cooperatively chooses the policy vector (t_i, τ_i, s_i) to maximize its domestic welfare. Welfare in country i is defined as the weighted sum of tax revenue in country i and private capital income. The private capital income of a representative resident in country i equals the return to her capital endowment, $r\bar{k}$, and the after-tax profit of both the domestic firm, $(1 - t_i)\pi_N^i$, and of the MNC headquartered in country i, Π_M^i. Thus, we assume that each MNC is fully owned by the resident of its HQ country.

We normalize the welfare weight of tax revenue to unity, whereas the welfare weight of private capital income is $\gamma \leq 1$. Our model therefore incorporates two different cases. In the first case, the welfare weight of private income coincides with that of tax revenue, $\gamma = 1$. In this case the government simply maximizes national income. A frequent interpretation of this case is that the government has access to an 'outside' lump-sum tax, and therefore pure revenue collection is not an argument to employ the policy instruments studied here. In the second case, there is also an outside tax, but this is distortionary and causes a fixed excess burden of taxation. In this case, $\gamma < 1$ and tax revenue has a higher weight than private income, as each dollar of corporate tax revenue can be used to reduce the outside, distortive tax.[20]

With these specifications, each country's tax problem is given by

$$\begin{aligned}
\max_{t_i,\tau_i,s_i} W_i &= t_i \left\{\pi_N^i(k_R^i) + f(k_i^i, q^i) - f_q(k_i^i, q^i)q^i - a_i^i\right. \\
&+ \left.(n-1)\left[f(k_i^j, q^j) - f_q(k_i^j, q^j)q^j - a_i^j\right]\right\} \\
&+ \tau_i \left\{f_q(k_i^i, q^i)q^i + (n-1)f_q(k_j^i, q^i)q^i + a_i^i + (n-1)a_j^i\right\} - s_i k_R^i \\
&+ \gamma \left\{r\bar{k} + (1-t_i)\pi_N^i(k_R^i) + \Pi_M^i\right\}. \qquad (11)
\end{aligned}$$

In (11), the investment levels $k_i^i(t_i, \tau_i, s_i)$ and $k_R^i(t_i, \tau_i, s_i)$ depend on all policy parameters. Note also that under symmetry the arm's-length price in country i turns into $p_i^i = f_q[k_i^i(t_i, \tau_i, s_i), q^i(t_i, \tau_i, s_i)]$. It is endogenous from the perspective of each government, because the average marginal productivity of technological quality will respond to changes in tax policy. On the other hand, the profit shifting terms $a_i^j(t_i, \tau_j)$ depend only on the difference between the statutory corporate tax rate and the tax rate on royalty income.

Each country i chooses its tax instruments (t_i, τ_i, s_i) to maximize (11), neglecting the impact of its policies on welfare in other countries.

3.1 Non-cooperative policy choices

Optimal statutory tax rate t_i. The first-order condition for the optimal statutory tax rate t_i in country i is

$$\begin{aligned}
\frac{\partial W_i}{\partial t_i} &= (1-\gamma)(\pi_N^i + B_1 - a_i^i) + (n-1)(B_2 - a_i^j) + [t_i + \gamma(1-t_i)]\frac{\partial \pi_N^i}{\partial k_R^i}\frac{\partial k_R^i}{\partial t_i} \\
&+ n t_i f_k \frac{\partial k_i}{\partial t_i} - n t_i q_i^i \left[f_{qk}\frac{\partial k_i}{\partial t_i} + f_{qq} q_k \frac{\partial k_R^i}{\partial t_i}\right] - t_i \frac{\partial a_i^i}{\partial t_i} - (n-1) t_i \frac{\partial a_i^j}{\partial t_i} \\
&+ n \tau_i q_i^i \left[f_{qk}\frac{\partial k_i}{\partial t_i} + f_{qq} q_k \frac{\partial k_R^i}{\partial t_i}\right] + n f_q \tau_i q_k^i \frac{\partial k_q^i}{\partial t_i} + \tau_i \frac{\partial a_i^i}{\partial s_i} - s_i \frac{\partial k_R^i}{\partial t_i} = 0, \qquad (12)
\end{aligned}$$

where $B_1 = f(k_i^i, q^i) - p_i q^i$ and $B_2 = f(k_i^j, q^j) - p_i q^j$ are the tax bases before profit shifting in affiliates in country i, and we have used the capital-market clearing condition (9) to simplify the first-order condition.

Since the focus of our analysis is on the tax treatment of R&D, it is sufficient for our purposes to show that the statutory corporate tax rate is positive in the optimum, i.e., $t_i^* > 0$. Hence, we evaluate (12) in a situation in which the government does not use any

[20] Treating the excess burden of 'outside' taxes as fixed can be justified by the fact that corporate tax revenue is only a small share of total tax revenue in OECD countries (typically 5-10%). See, e.g., Keen and Lahiri (1998) and Haufler et al. (2018) for further analyses using this assumption.

of its instruments and sets $t_i = \tau_i = s_i = 0$. Using symmetry, we then have $B_1 = B_2$ and $a_i^i = a_i^j = 0$. Hence, the first-order condition (12) simplifies to

$$\left.\frac{\partial W_i}{\partial t_i}\right|_{t_i=\tau_i=s_i=0} = (1-\gamma)\pi_N^i + (n-\gamma)B_1 + \gamma \frac{\partial \pi_N^i}{\partial k_R^i}\frac{\partial k_R^i}{\partial t_i} > 0. \tag{13}$$

Equation (13) shows that the profits earned by both national and multinational firms generate a first-order incentive for the government to tax these profits. The first term refers to the taxation of the domestic firm and leads to a welfare gain when $\gamma < 1$, as the tax redistributes from private income to tax revenue. Moreover, the corporate tax can also be used to tax *foreign* profits, as is shown by the second term in (13). This is because the tax falls not only on the domestic MNC, but also on the $(n-1)$ foreign-owned MNCs, whose reduction in private income does not enter country i's welfare function. Consequently, the optimal corporate tax rate will be positive even if $\gamma = 1$, and the transfer of resources from domestic residents to the government does not cause a welfare gain. Finally, by the third term, a higher corporate tax rate raises R&D investment, and thus increases the positive spillover effect on the national firm's profits. As all other tax distortions are of second order, and result only under a positive corporate tax rate, the optimal statutory tax rate t_i must be positive in the decentralized tax equilibrium.

Optimal patent box tax rate. Next, we consider the optimal tax rate on royalty income, τ_i. The first-order condition for this tax instrument is derived in Appendix A.2 and is given by

$$\begin{aligned}\frac{\partial W_i}{\partial \tau_i} \;=\; & (1-\gamma)n\left(f_q q^i + a^i\right) - t_i\frac{\partial a^i}{\partial \tau_i} + \tau_i n\frac{\partial a^i}{\partial \tau_i} + [t_i + \gamma(1-t_i)]\frac{\partial \pi_N^i}{\partial k_R^i}\frac{\partial k_R^i}{\partial \tau_i} + t_i f_k\frac{\partial k_i^i}{\partial \tau_i} \\ - \; & (t_i - \tau_i)nq^i\left[f_{qk}\frac{\partial k_i^i}{\partial \tau_i} + f_{qq}q_k\frac{\partial k_R^i}{\partial \tau_i}\right] + \tau_i n f_q q_k\frac{\partial k_R^i}{\partial \tau_i} - s_i\frac{\partial k_R^i}{\partial \tau_i} = 0. \end{aligned} \tag{14}$$

An increase in the royalty tax rate increases welfare by the direct revenue effect of the tax, net of the reduction in private income. This is the first term in the first line of equation (14). A higher tax rate on royalty income, however, reduces profit shifting via excessive royalty payments from all n producing affiliates to the R&D unit of the home-based MNC. These are the second and third terms in the first line. Via reduced R&D investment, an increase in the royalty tax also lowers the spillover effect on the national firm, reducing the revenue from taxing this firm's profit (the fourth term in the first line). Moreover, a higher royalty tax rate reduces capital investment in the domestic producing affiliate from (10c), as given by the last term in the first line.

Furthermore, the change in τ_i has ambiguous effects on the marginal productivity of R&D, f_q, and hence on the arm's-length price of royalty payments. The total change is weighted by the tax differential $t_i - \tau_i$, as seen by the first term in the second line of (14). The tax also reduces the quality of R&D, q^i. This lowers royalty payments of the domestic MNC to the patent box in country i, as seen in the second term in the second line. Finally, the induced reduction in R&D investment reduces the payments on R&D subsidies, as shown in the last term of (14).

Optimal R&D subsidies. Following equivalent steps, the first-order condition for R&D subsidies s_i results in

$$\begin{aligned}\frac{\partial W_i}{\partial s_i} &= -(1-\gamma)k_R^i + [t_i + \gamma(1-t_i)]\frac{\partial \pi_i^N}{\partial k_R^i}\frac{\partial k_R^i}{\partial s_i} + t_i f_k \frac{\partial k_i^i}{\partial s_i} \\ &- (t_i - \tau_i)nq^i\left[f_{qk}\frac{\partial k_i^i}{\partial s_i} + f_{qq}q_k\frac{\partial k_R^i}{\partial s_i}\right] + \tau_i n f_q q_k \frac{\partial k_R^i}{\partial s_i} - s_i\frac{\partial k_R^i}{\partial s_i} = 0. \qquad (15)\end{aligned}$$

An increase in the R&D subsidy reduces tax revenues and this leads to a negative direct effect on welfare, as given by the first term in the first line of (15). Via increased R&D investment, however, an increase in the R&D subsidy boosts profits of national firms, due to a larger positive spillover effect, and this leads to higher corporate tax revenues, as captured by the second term. Similarly, the higher R&D subsidy fosters capital investment in the domestic affiliate of the domestic MNC, increasing domestic corporate tax income by the last term in the first line. Like the royalty tax rate, the change in s_i has an ambiguous effect on the arm's-length price f_q, as given in the first term in the second line of (15). Moreover, the subsidy increases the R&D quality and boosts royalty payments of the domestic MNC, increasing revenue from the patent box in country i; the fifth term. Finally, the subsidy fosters R&D investment and leads to higher R&D subsidy expenditures for a given level of s_i, as shown by the last term in the second line of (15).

Note from the last two terms in (15) that a lower royalty tax rate τ_i (which implies that the positive fifth term becomes smaller in absolute value) implies a lower direct R&D subsidy, and hence a less negative last term, other things being equal. This shows that the implicit subsidization of R&D, which occurs through a lower patent box tax rate, is (partly) offset in the government's tax optimum by a lower R&D subsidy.

3.2 Tax policy mix in Nash equilibrium

We can now turn to the Nash equilibrium for the optimal tax policy mix. In our symmetric model, the first-order conditions in (12), (14) and (15) implicitly define an equilibrium where t_i^*, τ_i^* and s_i^* are simultaneously chosen. A symmetric Nash equilibrium exists in our model, if the welfare functions $W_i(t_i, \tau_i, s_i; t_j, \tau_j, s_j)\ \forall\ i, j\ (i \neq j)$ are continuous in the policies of both countries i and j, and strictly quasi-concave in the policies of country i. Continuity is guaranteed in our setting, because all components of W_i are continuous functions of the policy parameters in i and j. The first-order conditions in (12), (14) and (15) are too complex, however, to derive and prove the second-order conditions for the policy parameters t_i, τ_i and s_i. As is standard in the tax competition literature, we therefore have to assume that the sufficient second-order conditions for all tax policy choices are indeed met. Given the symmetry of our model, we can then infer that a symmetric Nash equilibrium must exist. While we cannot guarantee uniqueness of the Nash equilibrium, our focus in the following lies on the symmetric Nash equilibrium, even if further, asymmetric Nash equilibria should exist.

To study the properties of the symmetric Nash equilibrium, we combine the first-order conditions (14) and (15), using the fact that the patent box tax rate τ_i and the R&D subsidy have linearly dependent effects on the investment levels of both the R&D units and the producing affiliates [cf. eqs. (10b)-(10e)]. As shown in Appendix A.2, this leads to a simplified first-order condition for the royalty tax rate τ_i when the R&D subsidy is simultaneously optimized:

$$\frac{[(1-\gamma)+\frac{1}{n}]t_i - [(1-\gamma)+1]\tau_i}{\beta} + (1-\gamma) f_q (q^i - q_k k_R^i) = 0. \tag{16}$$

Eq. (16) has a straightforward interpretation. The optimal royalty tax τ_i, and hence the optimal design of the patent box, depends only on profit shifting considerations (the first term on the left-hand side), and on the existence of supernormal profits from R&D investment (the second term). The optimal royalty tax does not, however, depend on any real effects that τ_i has on R&D investment, or on the spillover on the host country's national firm. All the real effects on R&D and its externality on the domestic economy are instead captured by the optimal R&D subsidy s_i implicitly defined in (15). The optimal R&D subsidy trades off the direct revenue cost of R&D subsidies against the revenue increases that follow from the induced changes in R&D investment and total output production. In general, the sign of the optimal R&D subsidy is ambiguous. However, if the spillover effect on π_N^i [the second term in (15)] is sufficiently large, the optimal subsidy will be positive, $s_i^* > 0$.[21]

In the following, we discuss the central equation (16) for two different cases, depending on whether governments have access to an outside lump-sum tax ($\gamma = 1$), or only to distortive taxes ($\gamma < 1$).

Lump-sum taxes available. With $\gamma = 1$, the second term on the left-hand side of (16) vanishes.[22] The first term also simplifies and the optimal tax pattern (t_i, τ_i) reduces to

$$\frac{t_i^*}{\tau_i^*} = n > 1. \tag{17}$$

[21]Empirical studies suggest that the spillover effects are indeed large. Bloom et al. (2013, pp. 1383ff), for example, estimate the marginal social return on R&D to be on average twice as high as the marginal private return (58% to 20.8%).

[22]The second term would also vanish, if the production technology for innovation $q^i(k_R^i)$ is linear. In this case, $q^i = q_k k_R^i$ and there are no profits in the R&D sector.

Eq. (17) shows that the royalty tax rate τ_i is unambiguously *below* the statutory corporate tax rate t_i in the national tax optimum. Consequently, a patent box regime with a reduced rate on royalty income emerges under tax competition. The reason is that, by reducing τ_i, country i gives an incentive to *all* foreign affiliates of its home-based MNC to make excessive royalty payments to the R&D unit in country i. Hence, a marginal reduction in τ_i leads to an inframarginal inflow of additional tax base from abroad. Evaluated at $t_i = \tau_i$ there is, however, no first-order loss in tax revenue when the domestic affiliate of the home-based MNC also shifts some of its profits into the patent box. Therefore, in the tax optimum, $\tau_i^* < t_i^*$ must hold.

It is also straightforward to see from equation (17) that the tax discount on royalty income becomes larger when the number of foreign affiliates of the domestic MNC grows (i.e., when n rises). The more foreign affiliates a domestic MNC has, the more tax base can be gained from other countries by reducing the royalty tax rate τ_i, whereas the domestic tax revenue cost of lowering τ_i is unaffected by the parameter n. In this sense, the continued integration of MNCs in the global economy (i.e., an increasing number of affiliates n) increases the incentives for the domestic government to grant generous patent box regimes with low royalty tax rates τ_i.

No lump-sum taxes. We now turn to the general case where outside taxes are distortionary and $\gamma < 1$. When the R&D production function is concave ($q_{kk} < 0$), R&D investment generates supernormal profits and $q^i - q_k k_R^i > 0$ in the second term of (16). Hence there is now an incentive to tax royalty income at a *higher* rate, all else equal.

This can be seen in the special case where profit shifting is prohibitively expensive for the MNC ($\beta \to \infty$). In this case, the first term in (16) is zero and the second term of this equation is always positive. In combination with (15), we get

$$\left(\frac{1}{n f_q q_k} \frac{\partial W_i}{\partial \tau_i} + \frac{\partial W_i}{\partial s_i} \right) \Big|_{\beta \to \infty} = \frac{(1-\gamma)(q^i - q_k k_R^i)}{q_k} > 0. \tag{18}$$

The combination of a special tax rate τ_i on royalty income and a R&D subsidy s_i then jointly implements a tax system that taxes the economic rent from R&D without creating distortions on the intensive R&D investment margin. The subsidy ensures that the marginal return to k_R in (8b) remains undistorted, while the royalty tax confiscates all rents. The optimal royalty tax is then equal to one, and it must exceed the statutory tax rate t_i, which distorts the investment decisions of producing affiliates.

In the general case, the profit shifting effect [the first term in (16)] is present, however. As discussed above, this isolated effect works towards a lower tax on royalty income, as compared to the statutory tax rate t_i.[23] Therefore, with positive profits from R&D and profit shifting, the tax gap $t_i - \tau_i$ cannot be signed unambiguously. The easier is profit

[23] Note from the first term in (16) that $t_i^* > \tau_i^*$ also holds in the case of $\gamma < 1$.

shifting (the lower is β), the more likely is it that the profit shifting motive dominates the motive to tax the pure profits in the R&D sector. In the extreme case of costless profit shifting ($\beta \to 0$), the second term in (16) becomes negligible, only profit shifting matters, and the corporate tax rate must exceed the tax rate on royalties.

We summarize our results for the non-cooperative setting of tax policies in:

Proposition 1 *When countries compete in the set of tax policies (t_i, τ_i, s_i), the following holds:*

(i) The R&D subsidy s_i is the marginal instrument used to increase technological quality, and to internalize the spillover effects from R&D.

(ii) At the margin, the royalty tax rate τ_i is determined only by profit shifting considerations, and by the level of pure profits in the R&D sector.

(iii) When governments have access to lump-sum taxes ($\gamma = 1$), the optimal royalty tax rate is strictly below the statutory corporate tax rate, $\tau_i^* < t_i^*$.

(iv) The tax gap $(t_i^* - \tau_i^*)$ always increases in the number of MNC affiliates n. For $\gamma < 1$, the tax gap is also increasing in the ease of profit shifting (a fall in β).

Parts (*i*) and (*ii*) of Proposition 1 correspond to the fundamental *principle of targeting.* Both the R&D subsidy s_i and the royalty tax rate τ_i are able to increase real R&D activity, but the royalty tax rate has the additional affect of attracting inward profit shifting through excessive royalty payments. Therefore, when all policy instruments are simultaneously optimized, the R&D subsidy is directed exclusively at the R&D investment margin, whereas the royalty tax rate is focused on the MNC's profit shifting margin. This pattern suggests that the rising trend in R&D subsidies (see Appelt et al., 2019) is to be explained by a higher valuation of the technological improvements and their spillover effects that are induced by higher R&D subsidies, or by the reduced costs of providing such subsidies (for example, because of low interest rates on government debt).

On the other hand, the proliferation of patent box regimes shown in Table 1 can be explained by increased levels of MNCs' profit shifting (see Zucman, 2014, Figures 2 and 3). The feature of patent boxes to tax royalty payments at a rate below the statutory corporate tax rate emerges endogenously in our model, and it is isolated in the case where $\gamma = 1$ [see Proposition 1(*iii*)]. Finally, increased levels of profit shifting are at least partly caused by a growing number of MNCs that have a large number n of affiliates abroad, and that own valuable intangible assets that make it relatively easy to shift profits between countries (i.e., these MNCs face a low β). Indeed, the few available time series on MNCs and their affiliates, based on UNCTAD data, report that the number of MNCs almost tripled between 1995 and 2010 (from 38,500 to 104,000), and the total number of affiliates increased by a factor 3.5 from 251,500 to 892,000. The average number of affiliates per MNC hit a maximum of about 10 just before the financial crisis in 2007/08, see Jaworek and Kuzel (2015, p. 57 and Table 1). These trends are captured in part (*iv*) of Proposition 1.

4 Global welfare maximization

In this section, we derive the optimal coordinated policies when governments maximize global welfare. By doing so, we can compare the Pareto-efficient tax rules derived in this section with the tax competition equilibrium of Section 3 to identify strategic 'beggar-thy-neighbour' policies. We will maintain the assumption that all countries are symmetric and choose identical tax policies in the cooperative tax equilibrium.

Global welfare W_G is obtained by summing over the welfare levels of all countries, $W_G = nW_i$. This gives

$$\begin{aligned} W_G &= nt\left\{\pi_N^i(q^i) + f(k_i^i, q^i) - f_q(k_i^i, q^i)q^i - a_i^i + (n-1)\left[f(k_i^j, q^j) - f_q(k_i^j, q^j)q^j - a_i^j\right]\right\} \\ &+ n\tau\left\{nf_q(k_i^i, q^i)q^i + a^i\right\} - ns_i k_R^i + n\gamma\ \left\{r\bar{k} + (1-t_i)\pi_N^i(q^i) + \Pi_M^i\right\}, \end{aligned} \tag{19}$$

where investment levels $k_i^i(t,\tau,s)$ and $k_R^i(t,\tau,s)$ and R&D output $q^i(t,\tau,s)$ now depend on the coordinated tax instruments. Profit shifting $a_i^j(t,\tau)$ will still arise whenever the statutory tax rate and the royalty tax rate diverge in the coordinated tax equilibrium.

Globally optimal statutory tax rate. Differentiating (19) with respect to the statutory tax rate t, using capital market clearing from (9) and dividing by n leads to

$$\begin{aligned} \frac{1}{n}\frac{\partial W_G}{\partial t} &= (1-\gamma)\left[\pi_N^i + n\left(f(k_i^i, q^i) - p_i q^i - a_i\right)\right] + [t + \gamma(1-t)]\frac{\partial \bar{\pi}_N^i}{\partial k_R^i}\frac{\partial k_R^i}{\partial t} \\ &+ nt\left(f_k\frac{\partial k_i}{\partial t} - q^i\left[f_{qk}\frac{\partial k_i}{\partial t} + f_{qq}q_k\frac{\partial k_R^i}{\partial t}\right] - \frac{\partial a_i}{\partial t}\right) \\ &+ n\tau\left(q^i\left[f_{qk}\frac{\partial k_i}{\partial t} + f_{qq}q_k\frac{\partial k_q^i}{\partial t}\right] + f_q q_k\frac{\partial k_R^i}{\partial t} + \frac{\partial a_i}{\partial t}\right) - s\frac{\partial k_R^i}{\partial t} = 0. \end{aligned} \tag{20}$$

Again, it is sufficient to show that the optimal statutory tax rate is positive also under tax coordination. Evaluating the first-order condition (20) at $t = \tau = s = 0$ gives

$$\left.\frac{\partial W_G}{\partial t}\right|_{t=\tau=s=0} = n(1-\gamma)\left[\pi_N^i + n\left(f(k_i^i, q^i) - p_i q^i\right)\right] + n\gamma\frac{\partial \pi_N^i}{\partial k_R^i}\frac{\partial k_R^i}{\partial t} > 0. \tag{21}$$

Under policy coordination, there is no incentive to exploit the affiliates of foreign MNCs through a higher statutory tax rate, as is the case under unilaterally optimal tax policy [cf. the second term in (13)]. Nevertheless, the optimal coordinated statutory tax rate is always positive from the second effect, $t^* > 0$, even if $\gamma = 1$ and the first effect is zero.

Globally optimal patent box tax rate. The first-order condition for the coordinated royalty tax τ is derived in Appendix A.3 and is given by

$$\begin{aligned}\frac{1}{n}\frac{\partial W_G}{\partial \tau} &= n(1-\gamma)\left(f_q q^i + a_i\right) - n(t-\tau)\frac{\partial a_i}{\partial \tau} + [t+\gamma(1-t)]\frac{\partial \pi_N^i}{\partial q^i}\frac{\partial k_R^i}{\partial \tau} + ntf_k\frac{\partial k_i^i}{\partial \tau} \\ &- n(t-\tau)q^i\left[f_{qk}\frac{\partial k_i^i}{\partial \tau} + f_{qq}q_k\frac{\partial k_R^i}{\partial \tau}\right] + n\tau f_q q_k\frac{\partial k_R^i}{\partial \tau} - s\frac{\partial k_R^i}{\partial \tau} = 0. \qquad (22)\end{aligned}$$

In comparison to the non-coordinated setting of τ_i [eq. (14)], the critical difference lies in the second term in (22). The changes in tax base are now considered for *all* countries when a reduced royalty tax rate τ induces profit shifting from producing affiliates to the R&D unit. Starting from a patent box regime with a lower tax rate on royalty income ($t > \tau$), an increase in τ will therefore raise world welfare by this second effect (as $\partial a_i/\partial \tau < 0$).

Globally optimal R&D subsidy. Similarly, the first-order condition for the globally optimal R&D subsidy is

$$\begin{aligned}\frac{1}{n}\frac{\partial W_G}{\partial s} &= -(1-\gamma)k_R^i + [t+\gamma(1-t)]\frac{\partial \pi_N^i}{\partial k_R^i}\frac{\partial k_R^i}{\partial s} + ntf_k\frac{\partial k_i^i}{\partial s} \\ &- n(t-\tau)q^i\left[f_{qk}\frac{\partial k_i^i}{\partial s} + f_{qq}q_k\frac{\partial k_R^i}{\partial s}\right] + n\tau f_q q_k\frac{\partial k_R^i}{\partial s} - s\frac{\partial k_R^i}{\partial s} = 0. \qquad (23)\end{aligned}$$

The main difference to the optimality condition for the non-coordinated setting of R&D subsidies [eq. (15)] lies in the positive third effect in (23). For a rise in s, the positive corporate tax base effects in all n affiliates of an MNC will now be taken into account, thus internalizing the positive fiscal externality of R&D in one country on the remaining $(n-1)$ countries. Other things equal, the optimal coordinated R&D subsidy will therefore be higher than in the decentralized policy equilibrium.

The three first-order conditions (20), (22) and (23) implicitly define a coordinated tax equilibrium in three interdependent instruments. Again we assume that the second-order conditions are fulfilled and the coordinated tax instruments define a welfare maximum. We can again combine the first-order conditions (22) and (23), using the linear relationship in the comparative-static effects of the royalty tax and the R&D subsidy.[24] Appendix A.3 derives the simplified first-order condition for the coordinated royalty tax rate when the R&D subsidy is simultaneously chosen to maximize global welfare. This is

$$(2-\gamma)(t^* - \tau^*) + \beta(1-\gamma)f_q(q^i - q_k k_R^i) = 0, \qquad (24)$$

where, in comparison to eq. (16) above, all terms have been multiplied by β.

[24]The qualitative effects of changes in coordinated tax instruments on the capital market equilibrium can be inferred by substituting $n = 1$ in the results in Appendix A.1. This is because the effects of coordinated policy changes in a union of countries are analogous to policy changes in a closed economy.

If the government disposes of lump-sum taxes and $\gamma = 1$, the second term in (24) is again zero. In this case, it follows immediately that there is no reason to differentiate the statutory and the royalty tax rates, and the optimal coordinated policy implies $t^* = \tau^*$. The reason for this is straightforward. The only effect of reducing τ below t is to permit MNCs to shift profits from their producing affiliates to a tax-privileged patent box. Since this process incurs costs, profit shifting increases private after-tax income by less than it reduces global tax revenue. This can never be optimal when the welfare weight of tax revenue is equal to that of private profit income.

If outside lump-sum taxes do not exist and $\gamma < 1$, the second term in (24) is instead positive. This implies that the royalty tax rate will *exceed* the statutory tax rate under global policy coordination. The intuition for this result is familiar from our discussion of non-cooperative policy making in the previous section. The combination of a tax on the returns to R&D and a subsidy on R&D inputs gives governments the possibility to tax the profits in the R&D sector without distorting the MNC's decision to invest in R&D. This will lead to $\tau^* > t^*$, as the statutory tax rate, but not the royalty tax, distorts the MNC's investment decision in the production of the final good.

Interestingly, the negative tax gap $t^* - \tau^* < 0$ will now incentivize MNCs to *underinvoice* their intangibles so as to reduce their royalty income. Effectively, the profit shifting incentive is inverted and this constrains the extent to which the R&D profits can be confiscated by governments. This effect is captured by the term β in the second term of (24). Full economic integration ($\beta \to 0$) implies that MNCs can shift income between tax bases without costs. This makes it impossible to enforce a higher tax on royalty income and leads to $t^* = \tau^*$ in the coordinated optimum. Higher costs of profit shifting ($\beta > 0$) will constrain the possibilities of MNCs to work around the higher royalty tax and the coordinated tax differential $\tau^* - t^* > 0$ increases. If profit shifting becomes prohibitively expensive ($\beta \to \infty$), the royalty tax turns into a true lump-sum tax and the coordinated level of the royalty tax will be $\tau^* = 1$.

We summarize these results in:

Proposition 2 *When symmetric countries coordinate their tax policies (t, τ, s), the following holds:*

(i) When governments dispose of lump-sum taxes ($\gamma = 1$), the optimal royalty tax rate equals the statutory corporate tax rate, $\tau^* = t^*$.

(ii) If $\gamma < 1$, the optimal royalty tax exceeds the statutory corporate tax rate, $\tau^* > t^*$. The tax gap $\tau^* - t^*$ falls when the costs of profit shifting fall (a fall in β).

It follows from Proposition 2 that a patent box with $t^* - \tau^* > 0$ is never part of a coordinated, Pareto-optimal tax policy. A direct implication of this finding is that the emerging use of patent box regimes reduces global welfare. Putting Propositions 1 and 2 together, our analysis thus provides a theoretical underpinning for the claim that patent boxes are a "harmful form of tax competition", and that they should be replaced by "well-designed research and development tax credits" (Bloom et al., 2019, p. 171).

5 Mobile R&D units vs. mobile patents

In our benchmark model of the previous sections, we have assumed that the location of R&D units, and the location of patents from which royalty incomes are derived, are fixed in the HQ country of the respective MNC. In this section, we relax these assumptions. In doing so, we analyze the implications of imposing a 'nexus requirement' on the use of patent box regimes, as introduced by the OECD (2015). A nexus requirement stipulates that a reduced tax rate on royalty income is permissible only if the patent has been developed in the country that applies the preferential rate. However, the nexus requirement implies that countries have an incentive to compete for *mobile R&D units*, to attract inward profit shifting. Therefore, to analyze the effects of a nexus requirement, the location of R&D units must be endogenized. This scenario is covered in Section 5.1. In the absence of a nexus requirement, it will be possible to relocate patents after the R&D process has been completed. This scenario of *patent mobility* is analyzed in Section 5.2.[25]

5.1 International relocation of R&D units

With an endogenous location choice of R&D units, the game analyzed in this section has three stages. In the third stage, MNCs choose their investment levels in the production affiliates and in the R&D unit, as well as the transfer price, conditional on government policies *and* the location of the R&D unit. All these decisions remain structurally unchanged from the benchmark model in the previous section. In the second stage, each MNC decides in which country its R&D unit shall reside, depending on the policy vector of governments. In the first stage, countries therefore compete not only over FDI in production affiliates and the shifting of profits, but also over the location of the internationally mobile R&D unit. The game is solved by backward induction.

For simplicity, we assume in this section that there are only $n = 2$ countries. There are, however, a large number of MNCs in each of the two countries. If an MNC headquartered in country i decides to set up its R&D unit in the foreign country $j \neq i$, it incurs an agency cost which is captured by the general cost parameter θ.[26] How severe the agency conflict becomes when the R&D unit is separated from the HQ depends on the type α

[25]The empirical analysis of Schwab and Todtenhaupt (2019) introduces a similar distinction and shows that patent box regimes with a nexus requirement have very different effects from those without nexus.

[26]Dischinger et al. (2014) provide empirical evidence that HQ units of MNC's are substantially more profitable than foreign subsidiaries. This indicates that MNCs have non-tax reasons to locate profitable units, such as R&D units, in their HQ country.

of the MNC, which is uniformly distributed with support $\alpha \in [\underline{\alpha}, \overline{\alpha}]$. High-$\alpha$ types face higher agency costs than low-α types. The total MNC-specific agency costs of separating the MNC's R&D unit from its HQ are given by the product $\alpha\theta$.[27]

We extend the notation from our benchmark model by introducing an additional subscript $u \in \{i, j\}$ for the country in which the R&D unit is hosted. Hence, the global profits Π^i_{Mu} (before agency costs) of an optimally invested MNC with HQ in country i and the R&D unit in country u can be written as

$$
\begin{aligned}
& \max_{k^i_{iu}, k^i_{ju}, k^i_{Ru}, a^i_{iu}, a^i_{ju}} \Pi^i_{Mu} = \pi^i_{Ru} + \pi^i_{iu} + \pi^i_{ju} \\
= \; & (1 - t_i) f(k^i_{iu}, q^i_u) + (1 - t_j) f(k^i_{ju}, q^i_u) + (t_i - \tau_u)(p_i q^i_u + a^i_{iu}) - \frac{\beta}{2}(a^i_{iu})^2 \\
+ \; & (t_j - \tau_u)(p_j q^i_u + a^i_{ju}) - \frac{\beta}{2}(a^i_{ju})^2 - r[k^i_{iu} + k^i_{ju} + k^i_{Ru}] + s_u k^i_{Ru}, \qquad (25)
\end{aligned}
$$

where π^i_{ju} and π^i_{Ru} denote the profits of the production affiliate in country j and the R&D unit, respectively, of an MNC residing in country i and hosting its R&D unit in country $u \in \{i, j\}$. The tax rate τ_u is the patent box tax rate applicable in the R&D unit's host country u.

For the decision of where to locate the R&D unit, an MNC headquartered in country i compares its global after-tax profits with the R&D unit also in country i to the profits, including agency costs, of placing the R&D unit in country j (where $j \neq i$); that is it compares Π^i_{Mi} to $\Pi^i_{Mj} - \alpha^i\theta$. The pivotal MNC that is indifferent between locating its R&D unit in either country i or j is then defined by the cutoff value

$$
\hat{\alpha}^i = \frac{\Pi^i_{Mj} - \Pi^i_{Mi}}{\theta}. \qquad (26)
$$

MNCs of type $\alpha^i > \hat{\alpha}^i$ are too vulnerable to agency conflicts when locating their R&D unit away from the HQ, and will decide to host their R&D unit in the HQ country.

For a uniform distribution of types α, the number of R&D units in country i is given by $x^i = 1 - \hat{\alpha}^i + \hat{\alpha}^j$, where $\hat{\alpha}^i$ captures the share of MNCs headquartered in country i that locates their R&D unit in country j, and $\hat{\alpha}^j$ is the share of MNCs based in country j that place their R&D unit in country i. Appendix A.4 derives the effects of changes in country i's policy parameters on the pivotal type $\hat{\alpha}^i$. With symmetry, we get:

$$
\frac{d\hat{\alpha}^i}{dt_i} = 0, \qquad \frac{d\hat{\alpha}^i}{d\tau_i} > 0, \qquad \frac{d\hat{\alpha}^i}{ds_i} < 0. \qquad (27)
$$

An increase in the corporate tax rate in country i has no effect on $\hat{\alpha}^i$, because the tax does

[27] An alternative interpretation of the costs $\alpha\theta$ is that FDI and the setting up of a foreign affiliate require MNC-specific market entry costs (Arkolakis, 2010). These costs are heterogeneous and only low-cost types will become an MNC and set up a foreign affiliate (here: a foreign R&D unit).

not directly affect the profits of the R&D unit, and the induced interest rate changes are the same for R&D investments in countries i and j. A higher royalty tax rate τ_i reduces the return on R&D investment in country i. This induces more MNCs to locate their R&D unit in the other country j, increasing $\hat{\alpha}^i$. In contrast, a higher R&D subsidy in country i reduces the costs of R&D investment and makes country i more attractive for R&D units, reducing $\hat{\alpha}^i$. The effects of changes in the policy parameters of country j are qualitatively equivalent, and of opposite sign.

Appendix A.4 derives the optimal royalty tax rate τ_i, and optimal R&D subsidy s_i for this extended model.[28] We assume again that the second-order conditions are fulfilled, and the optimal tax choices define a symmetric Nash equilibrium. Combining the first-order conditions for τ_i and s_i leads to:

$$\frac{t_i^*}{\tau_i^*} = \frac{4 - 2\gamma - 4B/\theta}{3 - 2\gamma - 4B/\theta} - \beta \, \frac{f_q[q - q_k k_R]}{\tau_i} \frac{[2(1-\gamma) - 4B/\theta]}{(3 - 2\gamma - 4B/\theta)} \, , \tag{28}$$

where

$$B = \tau_i \pi_{Ri}^i - s_i k_R^i + 2k_R^i [t_i + \gamma(1 - t_i)] \frac{\partial \pi_N}{\partial k_R^i}$$

is the total net benefit of attracting an additional R&D unit. This net benefit equals the additional tax revenue from the profits of the R&D unit, net of payments for the R&D investment subsidy, and augmented by the effects of the positive externality that increased local R&D has on the profits of the national firm.

It is straightforward to compare eq. (28) to (16) in our benchmark model. For prohibitively high agency costs ($\theta \to \infty$), the first term in (28) corresponds to the first term in (16) when $n = 2$. Therefore, the basic motivation for taxing R&D profits at a rate below the corporate income tax remains unchanged in this extended setup. For finite values of θ the gap between t and τ is enlarged, however, due to the mobility of R&D units.[29] This is intuitive, because the profits of the R&D unit are taxed at the royalty tax rate τ. Therefore, international competition for R&D units will reduce the royalty tax rate τ in equilibrium, relative to the statutory corporate tax rate t.

The interpretation of the second term in (28) differs from that in our benchmark model. If $\gamma = 1$, this term becomes positive, and thus, *increases* the tax gap $t_i^* - \tau_i^*$. Intuitively, if $\gamma = 1$, taxing pure profits in the R&D unit of the *domestic* MNC does not increase welfare, but there is still an incentive to tax pure profits in R&D units of *foreign* MNCs. In order to earn such tax revenue, however, the foreign R&D units first need to be attracted, and this is done by reducing the patent box tax rate. If instead $\gamma < 1$, the

[28] The optimality condition for the statutory tax rate is extended from the benchmark model, as it includes the effects of t_i on the relocation of R&D units. However, this does not change our result that the corporate tax rate t_i will always be positive in the government's optimum. Therefore, we omit the first-order condition for the statutory tax rate from this extension.

[29] The parameter range for θ must be restricted to ensure that the denominator in both terms of (28) remains positive.

government also wants to tax economic profits in the R&D unit of the domestic MNC, calling for $\tau_i^* > t_i^*$, just as in our benchmark model. Then, the second term in (28) may turn negative, and therefore tend to raise τ_i^*. Finally, note from (28) that there is no effect on R&D investment at the margin. Consequently, with respect to the intensive investment margin, the targeting property for the patent box tax rate and the R&D subsidy, as identified in Proposition 1 (i) and (ii), remains in place.

We emphasize that the competition for R&D units (i.e., the extensive margin) in this setting occurs through both the patent box rate (as discussed above) and the R&D subsidy. The optimality condition for the R&D subsidy is (cf. Appendix A.4)

$$\begin{aligned}\frac{\partial W_i}{\partial s_i} &= -(1-\gamma)k_{Ri} + [t_i + \gamma(1-t_i)]\frac{\partial \pi_N^i}{\partial RD^i}\left(\frac{\partial k_{Ri}}{\partial s_i} - 2k_{Ri}\frac{\partial \hat{\alpha}^i}{\partial s_i}\right) + 2t_i f_k \frac{\partial k_i}{\partial s_i} + 2\tau_i f_q q_k^i \frac{\partial k_{Ri}}{\partial s_i} \\ &+ 2(\tau_i - t_i)q^i\left[f_{qk}\frac{\partial k_i}{\partial s_i} + f_{qq}q_k^i\frac{\partial k_R^i}{\partial s_i}\right] - s_i\frac{\partial k_{Ri}}{\partial s_i} - 2[\tau_i \pi_R^i - s_i k_{Ri}]\frac{\partial \hat{\alpha}^i}{\partial s_i} = 0. \qquad (29)\end{aligned}$$

The main difference to the benchmark case [eq. (15)] lies in the positive last term of eq. (29), which tends to raise the optimal R&D subsidy s_i^*. Therefore, the optimal R&D subsidy incorporates a strategic effect under this extension that is absent in the benchmark model. This strategic use of R&D subsidies to attract internationally mobile R&D units is consistent with recent findings in the empirical literature that MNCs relocate their R&D units to countries where they receive the highest subsidy (Knoll et al., 2019).[30]

5.2 International relocation of patents

In this section, we consider the alternative case where patents, after having been 'produced' in the R&D unit in the MNC's HQ country, can be relocated to another country, benefitting there from a reduced patent box tax rate. This corresponds to a setting where no internationally binding nexus requirements are introduced. In this analysis we assume, however, that the location of the R&D unit is fixed in the MNC's HQ country.

From an analytical perspective, this case has many parallels to the case where R&D units are internationally mobile, but there are also important differences. We restrict our analysis again to $n = 2$ countries. Hosting the patent in another country than the country where the development has occurred causes additional costs, which we capture with a general cost parameter θ_P. One important cost that may arise from the relocation of patents are 'exit taxes' that many countries levy when a patent that has been developed domestically is leaving the country. Our cost specification captures either the exit tax that has to be paid, or the extra administrative and legal efforts needed to bypass these exit taxes. Once again, we assume that MNCs are affected in different ways by these costs, for example because exit taxes are easier to quantify and to enforce in some sectors

[30]Our result also corresponds to the finding of strategic business-stealing competition in R&D subsidies; see, for example, Haaland and Kind (2008).

than in others. Therefore, we retain the individual cost parameter α, which is uniformly distributed with support $\alpha \in [\underline{\alpha}, \overline{\alpha}]$. Effective relocation costs of separating the patent from the R&D unit are then given by the MNC-specific costs $\alpha\theta_P$.

Once again, we have a three-stage game. In the third stage, all results on firms' choices remain unchanged, conditioned upon government policy *and* the location of the patent. In the second stage, the MNC decides in which country to place the patent, depending on the policy choices of governments. Hence, there is now international policy competition over the location of patents, in addition to the competition for FDI and for profit shifting.

The global profits Π^i_{Mu} of an MNC now carry a subscript u to specify the location of the patent in $u \in \{i, j\}$. With analogous notation for other variables, the MNC's maximization problem is

$$\begin{aligned}
\max_{k^i_{iu}, k^i_{ju}, k^i_{Ru}, a^i_{iu}, a^i_{ju}} \quad \Pi^i_{Mu} &= \pi^i_{Ru} + \pi^i_{iu} + \pi^i_{ju} \\
&= (1-t_i) f(k^i_{iu}, q^i_j) + (1-t_j) f(k^i_{ju}, q^i_u) + (t_i - \tau_u)(p_i q^i_u + a^i_{iu}) - \frac{\beta}{2}(a^i_{iu})^2 \\
&+ (t_j - \tau_u)(p_j q^i_u + a^i_{ju}) - \frac{\beta}{2}(a^i_{ju})^2 - r[k^i_{iu} + k^i_{ju} + k^i_{Ru}] - s_i k^i_{Ru},
\end{aligned} \tag{30}$$

where π^i_{ju} denotes profits in the production affiliate in country j of an MNC residing in country i and hosting its patent in country u. A critical difference to the case of mobile R&D units is that the R&D subsidy s_i in the last term is decided in the country where the R&D activity is located (i.e., in the HQ country of the MNC). Hence the R&D subsidy may now be granted by a different country than the country in which the patent is eventually placed and to which royalty income flows.

For the decision of where to locate the patent, an MNC headquartered in country i compares global after-tax profits with the patent located in its home country i to the ones, including relocation costs, of placing the patent in country j. In equilibrium, the cutoff value $\hat{\alpha}^i_P$ for an MNC headquartered in country i that is just indifferent between having its patent in country i or in country j is

$$\hat{\alpha}^i_P = \frac{\Pi^i_{Mj} - \Pi^i_{Mi}}{\theta_P}. \tag{31}$$

Formally, the analysis is fully analogous to that in the previous section. Using these analogies, Appendix A.5 derives the effects of all tax parameters on the MNC with the critical cost level $\hat{\alpha}^i_P$. Imposing symmetry yields:

$$\frac{d\hat{\alpha}^i_P}{dt_i} = 0, \qquad \frac{d\hat{\alpha}^i_P}{d\tau_i} > 0, \qquad \frac{d\hat{\alpha}^i_P}{ds_i} = 0. \tag{32}$$

The main difference to the case of mobile R&D units [eq. (27)] is that the R&D subsidy s_i has no effect on the pivotal MNC with patent relocation costs $\hat{\alpha}^i_P$. This is because the decision of where to locate the patent is made only *after* the MNC has collected R&D subsidies in its parent country.

Appendix A.5 derives the optimal royalty tax rate τ_i and the optimal R&D subsidy s_i for this problem. Combining these two first-order conditions leads to

$$\frac{t_i^*}{\tau_i^*} = \frac{4-2\gamma-4B_P/\theta_P}{3-2\gamma-4B_P/\theta_P} - \frac{\beta f_q\,[q^i-q_k k_R^i][2(1-\gamma)-4B_P/\theta_P]}{\tau_i(3-2\gamma-4B_P/\theta_P)} + \frac{4\beta f_q\, q_k k_R^i B_P/\theta_P}{\tau_i(3-2\gamma-4B_P/\theta_P)}, \tag{33}$$

where $B_P = \tau_i \pi_R^i$ are the benefits of attracting an additional patent. These are given by the revenues derived from taxing the profits of the R&D unit, which all accrue in the country where the patent is located.

It is then straightforward to compare the optimal patent box regime under the international mobility of R&D units in (28) with the patent box regime that results under patent mobility in (33). The first terms in both equations are identical in structure and will always imply a tax discount for R&D profits in the national optimum. However, these terms will generally differ quantitatively in the two regimes. They depend on the comparison of the net benefits of attracting an additional R&D unit vs. attracting an additional patent (the terms B vs. B_P), and on the relative mobility of these two tax bases (the terms θ vs. θ_P).

The net benefits B and B_P differ by two components. On the one hand, the costs of R&D subsidies $s_i k_R^i$ must be subtracted from the net benefit of attracting an additional R&D unit in (28), but there is no corresponding cost in the term B_P in (33). As discussed above, this is because patents are attracted *after* the R&D process is completed, and subsidies from the host country of the R&D unit have been collected. On the other hand, a positive spillover from R&D investment on the local economy will only result when the R&D unit is attracted, whereas no corresponding gain accrues from attracting an additional patent. This is because positive spillovers are associated with the R&D *process*, not with the possession of a patent. In general, the comparison of the terms B vs. B_P in (28) and (33) is therefore ambiguous. Note, however, that for $\tau = t$, only the R&D subsidy s is used to internalize the spillover externality *and* to mitigate investment distortions from the corporate tax. Consequently, at least in this benchmark case, the subsidy will likely exceed the marginal externality such that $B_P > B$.

At the same time, the relative mobility of the two internationally mobile tax bases, as measured by θ vs. θ_P, also matters. Arguably, the costs of relocating an R&D unit are higher for an MNC as compared to the costs of relocating a patent. This holds in particular when exit taxes for relocating patents to other countries are low, or cannot be properly enforced. This expectation is confirmed by the empirical results of Schwab and Todtenhaupt (2019). Consistent with our setup, they find that some R&D units move, if there is a nexus requirement in place, whereas R&D is conducted in the home affiliate, and only the patent relocates, if the nexus is not enforced. They also show that the externalities caused by mobile R&D units are far smaller than those caused by mobile patents. The main reason for these results seems to be that there are large agglomeration benefits from real R&D activity that is clustered in certain hubs, making R&D activity rather unresponsive to international tax differentials (high θ).

Overall, therefore, it is very likely that B/θ in (28) is smaller than B_P/θ_P in (33). This implies that the tax wedge $t^* - \tau^*$ will be larger under international patent mobility as compared to the international mobility of R&D units. In other words, mobile patents lead to more aggressive patent box regimes, as compared to mobile R&D units.

The second term in (33) is fully analogous to the second term in (28), and the discussion there applies. Finally, there is a positive third term in (33), which is not present in (28). As we have discussed in our benchmark model, the instrument to foster R&D is the R&D subsidy, whereas the patent box regime is primarily used to attract profit shifting. The lower tax rate on R&D profits, which corresponds to an implicit output subsidy, will be counteracted in the government's optimum by a less generous direct subsidy for R&D [cf. our discussion of eq. (15)]. This limits the effectiveness of a generous patent box regime when R&D units are mobile. The interaction between τ_i and s_i is also present when patents are internationally mobile, but in this case the reduced R&D subsidy that accompanies a lower patent box tax rate does not affect the mobility of the patent. In this sense, patent box regimes are more powerful under patent mobility than under the mobility of R&D units. Consequently, this effect reinforces the result that the tax gap $t^* - \tau^*$ is larger under patent mobility than under the mobility of R&D units.

To conclude our analysis of this case, we turn to the first-order condition for the optimal R&D subsidy. This is (cf. Appendix A.5)

$$\begin{aligned}\frac{\partial W}{\partial s_i} &= -(1-\gamma)k_R^i + [t_i + \gamma(1-t_i)]\frac{\partial \pi_N^i}{\partial RD^i}\frac{\partial k_R^i}{\partial s_i} + 2(\tau_i - t_i)q^i\left[f_{qk}\frac{\partial k_i}{\partial s_i} + f_{qq}q_k^i\frac{\partial k_R^i}{\partial s_i}\right] \\ &+ \ 2t_i f_k \frac{\partial k_i}{\partial s_i} + 2\tau_i f_q q_k^i \frac{\partial k_R^i}{\partial s_i} - s_i\frac{\partial k_R^i}{\partial s_i} = 0. \qquad (34)\end{aligned}$$

Comparing (34) with the globally efficient R&D subsidy in (23) shows that these are structurally identical. The only difference is that the optimal level of τ_i in (34) is smaller than in the coordinated case in (23). From the interaction of τ_i and s_i discussed above, this implies a lower subsidy s_i in the tax competition equilibrium, as compared to the globally optimal policies in (23). However, different from the case of internationally mobile units [eq. (29)], there is no incentive to strategically increase R&D subsidies, as these are powerless in attracting internationally mobile patents. Therefore, R&D subsidies will likely be set more efficiently under patent mobility, as compared to the case of internationally mobile R&D units. We summarize our results in this section as follows:

Proposition 3 *Comparing optimal policies under internationally mobile R&D units [eq.* (28)*] and under international patent mobility [eq.* (33)*] the following holds:*

(i) The patent box tax rate is strategically used for competition for R&D units and patents on the extensive margin, but it does not affect marginal investment per R&D unit. Thus, the targeting properties for the intensive margin [Proposition 1(i) and (ii)] carry over to mobile R&D units and mobile patents.

(ii) If the condition $B/\theta < B_P/\theta_P$ holds, then the tax gap $t^* - \tau^*$ will be larger, and patent box regimes are more aggressive, when patents are internationally mobile, as compared to internationally mobile R&D units.

(iii) Direct R&D subsidies are strategically increased under international mobility of R&D units, but not under international patent mobility.

Applying our results to the nexus debate is then straightforward. In Proposition 3(*ii*), we have summarized that patent mobility will lead to more aggressive reductions in the profit tax rate on R&D units, as compared to the international mobility of R&D units. Moreover, we know from Proposition 2 that any preferential tax treatment for royalty income is inefficient from a global welfare perspective. Therefore, our results for the structure of patent box regimes support international arrangements that stipulate such a nexus requirement, as this policy will likely curb a mutually destructive *race to the bottom* in the setting of patent box tax rates.

However, as shown in Proposition 3(*iii*), R&D subsidies will tend to be inefficiently high when a nexus requirement is introduced and each country is trying to attract internationally mobile R&D units. In a wider normative perspective that also incorporates direct policy tools to foster R&D, it is thus no longer clear that a nexus requirement leads to a superior allocation. From a positive perspective, we would expect that the OECD's policy decision in favor of the nexus approach leads to a continued rise in the R&D subsidies offered by competing countries.

6 Discussion and extensions

In this section we briefly discuss some further differences between patent box regimes and direct R&D subsidies in promoting innovative activities.

R&D as a dynamic and stochastic process. Our analysis has treated investments in the R&D sector as a static activity with deterministic results. In reality, R&D activities are long-run decisions with stochastic outcomes.[31] This leads to additional differences between R&D subsidies and reduced royalty tax rates, which have not been covered so far. First R&D subsidies are paid irrespective of whether the R&D activity leads to a

[31] Davies et al. (2020), for example, document that only about 50% of R&D activities are 'successful', in the sense that their patent application is granted.

successful outcome, whereas a reduced royalty tax rate benefits the investor only if the R&D activity is successful in generating patents, for which royalty income is earned. Risk-inverse investors will therefore prefer a R&D subsidy that is equal in expected value to a reduced tax on royalty income.

Second, R&D subsidies are a *front-end measure* that immediately reduces the costs of R&D investments. In contrast, reduced taxes on royalty income are a *back-end measure* that becomes effective only when the R&D investment has been transformed into output. This difference in the timing of government support is critical when investors are liquidity-constrained. They will then prefer an R&D tax credit of equal discounted value, even if they are risk-neutral. This more detailed treatment of the R&D process will, however, not affect the profit shifting incentives that we identified earlier.

Patents versus technological improvements. Our analysis has assumed that patented activities are always associated with (some) technological improvements and increased economic activity. These beneficial effects of patents are in line with some of the empirical evidence (Bradley et al., 2015; Koethenbuerger et al., 2018; Chen et al., 2020). There is, however, also evidence that patent quality decreases (Bornemann et al., 2020), or that patents are simply relocated in response to patent boxes (Gaessler et al., 2019). More generally, there is a lively debate on 'bad patents', which do not lead to real innovations, but merely serve to establish property rights.[32] Such patents have little or no positive effects on economic activity, nor positive spillovers on other sectors. Indeed, as it is the granting of a patent that triggers preferential tax treatment under most existing patent box schemes (see Table 1), firms have an incentive to submit patent applications, even if they do not represent technological improvements. Hence, patent box regimes can be expected to contribute to the practice of patenting 'marginal innovations'.

If such unproductive patenting is introduced in our model, the impact of patents on production and its spillover effects are reduced or even disappear. The incentives to attract paper profits from other countries via patent boxes remain, however. Note also that 'bad patents' are less of a problem for direct R&D subsidies or R&D tax credits, because these promote the R&D process itself, rather than the patent obtained as an outcome. Therefore, if firms can decide between developing more productive and more costly patents versus low-cost, low-productivity patents, patent box regimes would attract a larger share of the latter type, in comparison to R&D tax credits.

In sum, both of these extensions imply that R&D tax credits are likely to be the preferred instrument for fostering innovation, in comparison to a patent box regime with reduced tax rates on intellectual property.[33] Consequently, these considerations strengthen

[32]Bessen and Meurer (2008) provide an early overview on this debate. An infamous example is the patent on the 'peanut-butter-and-jelly-sandwich', held by Smucker Fruit Processing Co. (Bessen and Meurer, 2008, p. 3).

[33]See Klemens (2016) for a related discussion that questions the suitability of patent box regimes as a

our finding that patent box regimes are primarily a beggar-thy-neighbor policy that is targeted at attracting international profit shifting.

7 Conclusions

Patent box regimes offer a reduced tax rate to business on their IP-related income. In this paper, we have analyzed the effects of such special tax regimes when governments can simultaneously choose optimal R&D subsidies. In this setting, the different policy instruments take on specialized roles. The R&D subsidy operates as the marginal policy instrument to promote real R&D activity, whereas the special tax rate on royalty income is used to attract profit shifting by MNCs. We have shown that patent box regimes with a preferential tax rate on royalty income emerge endogenously when countries that host large, well-integrated MNCs non-cooperatively choose their tax policies. In contrast, patent box regimes are never optimal under coordinated policy setting.

From a policy perspective, these results indicate that patent boxes can indeed be seen as a measure of 'harmful tax competition' (Bloom et al., 2019, p. 171). They are thus fundamentally similar to the harmful tax regimes that have been identified, and abolished, in OECD countries in the 1990s (see OECD, 1998; European Communities, 1998). Our analysis provides a theoretical rationale for the nexus approach agreed upon by OECD countries, as the nexus requirement prevents designs of patent box regimes that aim most aggressively at attracting foreign profit tax bases. However, the nexus approach is also likely to lead to a *race to the top* in the setting of R&D subsidies, in order to attract internationally mobile R&D units. In sum, our findings suggest that abolishing patent box regimes altogether would be the best policy alternative.

Our results have implications for empirical studies on patent box regimes and related policies that reduce the tax rate on IP-related business income. In order to evaluate whether these measures do indeed foster innovation, it is essential to control for simultaneous policies that directly subsidize R&D. Since R&D subsidies have increased substantially in recent years, neglecting these policies may falsely attribute increased innovative activity to the rise of patent box regimes. The results of Boesenberg and Egger (2017) suggest that patent box regimes may cease to have any stimulating effects on patenting activity when comprehensive measures of direct R&D subsidies are incorporated in the regression analysis. Our analysis offers a theoretical rationale for their findings. Finally, some additional testable hypotheses emerge from our analysis. In particular, the generosity of the patent box regime in a particular country is expected to increase in the connectedness (i.e., the number of affiliates) of this country's MNCs, and in the ease (the inverse of costs) with which profit shifting to the patent box occurs.

measure to foster R&D.

A Appendix

A.1 Comparative static results

The market clearing condition (9) can be restated as

$$\sum_u \sum_m k_m^u + \sum_u k_R^u = n\bar{k}, \tag{A.1}$$

where the superscript u indicates the MNC to which the affiliates belongs and the subscript m indicates the country in which a production affiliate is located.

The first-order conditions for capital inputs k_m^u and k_R^u in (8a) and (8b) are equal to

$$(1-t_m) f_k(k_m^u, q^u) - r = 0, \tag{A.2}$$

$$(1-\tau_u)\sum_m f_q(k_m^u, q^u) q_k^u - (r - s_u) = 0. \tag{A.3}$$

Totally differentiating the first-order conditions and using symmetry gives

$$dk_m^u = -\frac{f_{kq} q_k^u}{f_{kk}}\, dk_R^u + \frac{f_k}{(1-t_m) f_{kk}}\, dt_m + \frac{dr}{(1-t_m) f_{kk}}. \tag{A.4}$$

$$dk_R^u = -\frac{(1-\tau) f_k\, (f_{kq} q_k)}{(1-t) A f_{kk}} \sum dt_m + \frac{n f_q q_k}{A} d\tau_u - \frac{ds_u}{A} + \frac{(1-t) f_{kk} - (1-\tau) n (f_{kq} q_k)}{(1-t) A f_{kk}} dr, \tag{A.5}$$

where

$$A = \frac{(1-\tau) n}{f_{kk}} \left[f_{kk} \left(f_{qq} q_k^2 + f_q q_{kk} \right) - (f_{kq} q_k)^2 \right] < 0 \tag{A.6}$$

because $f_{kk}\left(f_{qq} q_k^2 + f_q q_{kk}\right) - (f_{kq} q_k)^2 > 0$ is implied by the second-order conditions for optimal MNC behavior.

Substituting (A.5) into (A.4), we can derive changes in capital inputs in production affiliates in response to changes in tax-policy variables and the interest rate. Collecting terms results in

$$\begin{aligned} dk_m^u &= \frac{(1-\tau) f_k f_{kk} (f_{kq} q_k)^2}{(1-t) A f_{kk}^2} \sum_m dt_m + \frac{f_k A f_{kk}}{(1-t) A f_{kk}^2} dt_m - \frac{(1-t) f_{kk} n f_q f_{kq} q_k^2}{(1-t) A f_{kk}^2} d\tau_u \\ &+ \frac{(1-t) f_{kk} f_{kq} q_k}{(1-t) A f_{kk}^2} ds_u - \frac{(1-t) f_{kk} (f_{kq} q_k) - (1-\tau) n (f_{kq} q_k)^2 - A f_{kk}}{(1-t) A f_{kk}^2} dr. \end{aligned} \tag{A.7}$$

From (A.5) and (A.7) it follows that the direct effects of changes in royalty tax rate τ_u and the R&D subsidy s_u are linearly dependent for a given interest rate ($dr = 0$). Hence

$$\left.\frac{dk_R^u}{d\tau_u}\right|_{dr=0} = -n f_q q_k \left.\frac{dk_R^u}{ds_u}\right|_{dr=0} \quad \text{and} \quad \left.\frac{dk_m^u}{d\tau_u}\right|_{dr=0} = -n f_q q_k \left.\frac{dk_m^u}{ds_u}\right|_{dr=0}. \tag{A.8}$$

Totally differentiating the market clearing condition (A.1) and inserting the expressions from (A.5) and (A.7) yields

$$\begin{aligned} dr &= -\frac{\{(1-\tau)nf_k(f_{kq}q_k)^2 + f_k A f_{kk} - (1-\tau)nf_k f_{kk}(f_{kq}q_k)\}}{\Delta} \sum_m dt_m \\ &+ \frac{(1-t)f_{kk}[f_{kq}q_k - f_{kk}]}{\Delta} \left\{ nf_q q_k \sum d\tau_u - \sum ds_u \right\}, \end{aligned} \tag{A.9}$$

where $\Delta \equiv n\left[(1-\tau)n(f_{kq}q_k)^2 + Af_{kk} - (1-\tau)nf_{kk}(f_{kq}q_k) + (1-t)f_{kk}^2\right] > 0$.

Since $A < 0$, it holds that $\frac{dr}{dt_m} < 0$, $\frac{dr}{d\tau_u} < 0$, and $\frac{dr}{ds_u} > 0$. It also follows that the induced changes in the interest rate caused by changes in either τ_i or s_i are linearly dependent. In conjunction with (A.8), we can thus conclude that

$$\frac{dk_R^u}{d\tau_u} = -nf_q q_k \frac{dk_R^u}{ds_u} \quad \text{and} \quad \frac{dk_m^u}{d\tau_u} = -nf_q q_k \frac{dk_m^u}{ds_u}. \tag{A.10}$$

Inserting the interest rate effects (A.9) in (A.5) and collecting terms gives the following comparative-static effects on capital investment in R&D:

$$\begin{aligned} \frac{dk_R^u}{ds_u} &= -\frac{(n-1)\left\{n(1-\tau)(f_{kq}q_k)^2 - n(1-\tau)f_{kk}(f_{kq}q_k) + (1-t)f_{kk}^2\right\}}{nA\Delta} \\ &- \frac{f_{kk}[nA - (1-t)f_{kq}q_k]}{nA\Delta} > 0, \end{aligned} \tag{A.11}$$

where Δ is given in (A.9). From (A.10) then follows $dk_R^u/d\tau_u < 0$. Finally, we have

$$\frac{dk_R^u}{dt_m} = -\frac{f_k\left\{n(1-\tau)(f_{kq}q_k)^2 + Af_{kk}\right\}}{nA\Delta} > 0. \tag{A.12}$$

Inserting the interest rate effects (A.9) into (A.7), the comparative-static effects on capital investment in final-good production are

$$\begin{aligned} \frac{dk_m^u}{ds_u} &= \frac{(n-1)\left\{n(1-\tau)(f_{kq}q_k)^3 + Af_{kk}(f_{kq}q_k) - n(1-\tau)f_{kk}(f_{kq}q_k)^2 + (1-t)f_{kk}^2(f_{kq}q_k)\right\}}{nAf_{kk}\Delta} \\ &+ \frac{Af_{kk} + (1-t)(f_{kq}q_k)^2}{nA\Delta} > 0, \end{aligned} \tag{A.13}$$

as long as the number of countries/affiliates n is sufficiently large to overcompensate the two terms in the second line. From (A.10) then follows $dk_m^u/d\tau_u < 0$.

An increase in the corporate tax rate of another country $v \neq m$ fosters capital investment in production affiliates in country m from

$$\frac{dk_m^u}{dt_v} = \frac{f_k\left\{n(1-\tau)(f_{kq}q_k)\left[(1-t)(f_{kq}q_k) - A(f_{kq}q_k) + Af_{kk}\right] + (1-t)Af_{kk}(f_{kq}q_k) - A^2 f_{kk}\right\}}{nAf_{kk}\Delta} > 0. \tag{A.14}$$

Finally, an increase in t_m has a negative effect on capital in production affiliates in country m:

$$\frac{dk_m^u}{dt_m}=\frac{(n-1)f_k\left\{A+(1-t)\frac{n}{n-1}f_{kk}-[n(1-\tau)-\frac{1-t}{n-1}]f_{kq}q_k+n(1-\tau)(f_{kq}q_k)^2\frac{A+\frac{1-t}{n-1}}{Af_{kk}}\right\}}{n\Delta}<0, \tag{A.15}$$

as long as the number of countries n is sufficiently large.

A.2 Optimal R&D taxes under policy competition

Optimal royalty tax rate. Differentiating national welfare W_i in (11) with respect to τ_i leads to the first-order condition

$$\begin{aligned}
\frac{\partial W_i}{\partial \tau_i} &= (1-\gamma)[f_q^i q^i+(n-1)f_q^j q^i+a_i^i+(n-1)a_j^i]-t_i\frac{\partial a_i^i}{\partial \tau_i}+\tau_i\left[\frac{\partial a_i^i}{\partial \tau_i}+(n-1)\frac{\partial a_j^i}{\partial \tau_i}\right]+t_i f_k\frac{\partial k_i^i}{\partial \tau_i}\\
&+ t_i\left(f_q q_k\frac{\partial k_R^i}{\partial \tau_i}-\left[f_{qk}\frac{\partial k_i^i}{\partial \tau_i}q^i+f_{qq}q_k\frac{\partial k_R^i}{\partial \tau_i}q^i+f_q q_k\frac{\partial k_R^i}{\partial \tau_i}\right]-(n-1)\left[f_{qk}\frac{\partial k_i^i}{\partial \tau_i}q^j+f_{qq}q_k\frac{\partial k_R^i}{\partial \tau_i}q^j\right]\right)\\
&+ \tau_i\left(\left[f_{qk}\frac{\partial k_i^i}{\partial \tau_i}q^i+f_{qq}q_k\frac{\partial k_R^i}{\partial \tau_i}q^i+f_q^i q_k\frac{\partial k_R^i}{\partial \tau_i}\right]+(n-1)\left[f_{qk}\frac{\partial k_j^i}{\partial \tau_i}q^i+f_{qq}q_k\frac{\partial k_R^i}{\partial \tau_i}q^i+f_q^j q_k\frac{\partial k_R^i}{\partial \tau_i}\right]\right)\\
&+ \gamma\left[\bar{k}-\left(k_i^i+(n-1)k_j^i+k_R^i\right)\right]\frac{\partial r}{\partial \tau_i}+[t_i+\gamma(1-t_i)]\frac{\partial \pi_N^i}{\partial k_R^i}\frac{\partial k_R^i}{\partial \tau_i}-s_i\frac{\partial k_R^i}{\partial \tau_i}=0. \qquad \text{(A.16)}
\end{aligned}$$

The term in squared brackets in the last line equals zero from the capital market clearing condition (9). Imposing symmetry and collecting terms then gives (14) in the main text.

Using equation (A.10) in (14), the FOC for τ_i can be rewritten as

$$\begin{aligned}
\frac{\partial W_i}{\partial \tau_i} &= (1-\gamma)n(f_q q^i+a_i^i)-t_i\frac{\partial a_i^i}{\partial \tau_i}+n\tau_i\frac{\partial a_i^i}{\partial \tau_i}-nf_q q_k[t_i+\gamma(1-t_i)]\frac{\partial \pi_N^i}{\partial k_R^i}\frac{\partial k_R^i}{\partial s_i} \qquad \text{(A.17)}\\
&- nf_q q_k\left\{t_i f_k\frac{\partial k_i^i}{\partial s_i}-(t_i-\tau_i)nq^i\left[f_{qk}\frac{\partial k_i^i}{\partial s_i}+f_{qq}q_k\frac{\partial k_R^i}{\partial s_i}\right]+\tau_i n f_q q_k\frac{\partial k_R^i}{\partial s_i}-s_i\frac{\partial k_R^i}{\partial s_i}\right\}=0.
\end{aligned}$$

Optimal R&D subsidy. Differentiating W_i in (11) with respect to s_i gives

$$\begin{aligned}
\frac{\partial W_i}{\partial s_i} &= -(1-\gamma)k_R^i+t_i f_k\frac{\partial k_i^i}{\partial s_i} \qquad \text{(A.18)}\\
&+ t_i\left(f_q q_k\frac{\partial k_R^i}{\partial s_i}-\left[f_{qk}\frac{\partial k_i^i}{\partial s_i}q^i+f_{qq}q_k\frac{\partial k_R^i}{\partial s_i}q^i+f_q q_k\frac{\partial k_R^i}{\partial s_i}\right]-(n-1)\left[f_{qk}\frac{\partial k_i^i}{\partial s_i}q^j+f_{qq}q_k\frac{\partial k_R^i}{\partial s_i}q^j\right]\right)\\
&+ \tau_i\left(\left[f_{qk}\frac{\partial k_i^i}{\partial s_i}q^i+f_{qq}q_k\frac{\partial k_R^i}{\partial s_i}q^i+f_q^i q_k\frac{\partial k_R^i}{\partial s_i}\right]+(n-1)\left[f_{qk}\frac{\partial k_j^i}{\partial s_i}q^i+f_{qq}q_k\frac{\partial k_R^i}{\partial s_i}q^i+f_q^j q_k\frac{\partial k_R^i}{\partial s_i}\right]\right)\\
&+ \gamma\left[\bar{k}-\left(k_i^i+(n-1)k_j^i+k_R^i\right)\right]\frac{\partial r}{\partial s_i}+[t_i+\gamma(1-t_i)]\frac{\partial \pi_N^i}{\partial k_R^i}\frac{\partial k_R^i}{\partial s_i}-s_i\frac{\partial k_R^i}{\partial s_i}=0.
\end{aligned}$$

Using capital market clearing from (9), applying symmetry and collecting terms gives the first-order condition (15) in the main text.

Inserting (15) into the rearranged first-order condition for τ_i in (A.17) results in

$$\frac{\partial W_i}{\partial \tau_i}\Big|_{s=s^*} = (1-\gamma)n(f_q[q^i - q_k k_R^i] + a_i^i) - t_i \frac{\partial a_i^i}{\partial \tau_i} + n\tau_i \frac{\partial a_i^i}{\partial \tau_i} = 0. \tag{A.19}$$

Using $\partial a_i^i/\partial t_i = -\partial a_i^i/\partial \tau_i = 1/\beta$ from the optimal profit shifting function (8c) leads to the optimality condition in equation (16) of the main text.

A.3 Optimal R&D taxes under policy coordination

Differentiating global welfare W_G in (19) for the simultaneous choice of the tax rate τ in all countries i leads to the first-order condition

$$\begin{aligned}
\frac{\partial W_G}{\partial \tau} &= n\Big\{(1-\gamma)n[f_q q^i + a_i] + [t+\gamma(1-t)]\frac{\partial \pi_N^i}{\partial k_R^i}\frac{\partial k_R^i}{\partial \tau} + nt\left(f_k \frac{\partial k_i^i}{\partial \tau} + f_q q_k \frac{\partial k_R^i}{\partial \tau}\right) \\
&- nt\left[f_{qk}\frac{\partial k_i^i}{\partial \tau}q^i + f_{qq}q_k \frac{\partial k_R^i}{\partial \tau}q^i + f_q q_k \frac{\partial k_R^i}{\partial \tau} + \frac{\partial a_i}{\partial \tau}\right] \\
&+ n\tau\left[f_{qk}\frac{\partial k_i^i}{\partial \tau}q^i + f_{qq}q_k \frac{\partial k_R^i}{\partial \tau}q^i + f_q q_k \frac{\partial k_R^i}{\partial \tau} + \frac{\partial a_i}{\partial \tau}\right] - s\frac{\partial k_R^i}{\partial \tau} = 0,
\end{aligned}$$

where capital market clearing [eq. (9)] has been used. Collecting terms leads to eq. (22) in the main text.

Applying (A.10), condition (22) transforms into

$$\begin{aligned}
\frac{1}{n}\frac{\partial W_G}{\partial \tau} &= n(1-\gamma)\left(f_q q^i + a_i\right) - n(t-\tau)\frac{\partial a_i}{\partial \tau} - nf_q q_k \Bigg\{[t+\gamma(1-t)]\frac{\partial \pi_N^i}{\partial k_R^i}\frac{\partial k_R^i}{\partial s} \qquad (A.20) \\
&+ ntf_k \frac{\partial k_i^i}{\partial s} - n(t-\tau)q^i\left[f_{qk}\frac{\partial k_i^i}{\partial s} + f_{qq}q_k \frac{\partial k_R^i}{\partial s}\right] + n\tau f_q q_k \frac{\partial k_R^i}{\partial s} - s\frac{\partial k_R^i}{\partial s}\Bigg\} = 0.
\end{aligned}$$

Differentiating global welfare W_G in (19) with respect to the R&D subsidy s gives

$$\begin{aligned}
\frac{\partial W_G}{\partial s} &= n\Big\{-(1-\gamma)k_R^i + nt\left(f_k \frac{\partial k_i^i}{\partial s} + f_q q_k \frac{\partial k_R^i}{\partial s} - \left[f_{qk}\frac{\partial k_i^i}{\partial s}q^i + f_{qq}q_k \frac{\partial k_R^i}{\partial s}q^i + f_q q_k \frac{\partial k_R^i}{\partial s}\right]\right) \\
&+ n\tau\left[f_{qk}\frac{\partial k_i^i}{\partial s}q^i + f_{qq}q_k \frac{\partial k_R^i}{\partial s}q^i + f_q q_k \frac{\partial k_R^i}{\partial s}\right] - s\frac{\partial k_R^i}{\partial s} + [t+\gamma(1-t)]\frac{\partial \pi_N^i}{\partial k_R^i}\frac{\partial k_R^i}{\partial s}, \qquad (A.21)
\end{aligned}$$

where the interest rate terms drop out from capital market clearing. Collecting terms and dividing by n then leads to (23) in the main text.

Finally, inserting (23) into the rearranged FOC for τ_i in (A.20) results in

$$\frac{\partial W_G}{\partial \tau}\Big|_{s=s^*} = n\left\{(1-\gamma)f_q[q^i - q_k k_R^i] + (2-\gamma)a_i\right\} = 0, \tag{A.22}$$

where we made use of $-(t-\tau)(\partial a_i/\partial \tau) = a_i$ from (8c). Solving this expression for a_i and using $a_i = (t-\tau)/\beta$ gives eq. (24) of the main text.

A.4 Location decision of the R&D unit

Applying the Envelope theorem to eq. (25) gives

$$
\begin{aligned}
\frac{\partial \Pi^i_{Mi}}{\partial t_i} &= -f(k^i_{ii}, q^i_i) + p_i q^i_i + a^i_{ii} - [k^i_{ii} + k^i_{ji} + k^i_{Ri}]\frac{\partial r}{\partial t_i}, \\
\frac{\partial \Pi^i_{Mj}}{\partial t_i} &= -f(k^i_{ij}, q^i_j) + p_i q^i_j + a^i_{ij} - [k^i_{ij} + k^i_{jj} + k^i_{Rj}]\frac{\partial r}{\partial t_i}, \\
\frac{\partial \Pi^i_{Mi}}{\partial \tau_i} &= -[p_i q^i_i + p_j q^i_i] - [a^i_{ii} + a^i_{ji}] - [k^i_{ii} + k^i_{ji} + k^i_{Ri}]\frac{\partial r}{\partial \tau_i}, \\
\frac{\partial \Pi^i_{Mj}}{\partial \tau_i} &= -[k^i_{ij} + k^i_{jj} + k^i_{Rj}]\frac{\partial r}{\partial \tau_i}, \\
\frac{\partial \Pi^i_{Mi}}{\partial s_i} &= k^i_{Ri} - [k^i_{ii} + k^i_{ji} + k^i_{Ri}]\frac{\partial r}{\partial s_i}, \\
\frac{\partial \Pi^i_{Mj}}{\partial s_i} &= -[k^i_{ij} + k^i_{jj} + k^i_{Rj}]\frac{\partial r}{\partial s_i}.
\end{aligned}
$$

The comparative-static effects of policy changes in country i on the location decision of R&D units can then be obtained by differentiating (26). This yields

$$
\begin{aligned}
\frac{d\hat{\alpha}^i}{dt_i} &= -\frac{[f(k^i_{ij}, q^i_j) - p_i q^i_j] - [f(k^i_{ii}, q^i_i) - p_i q^i_i] - [a^i_{ij} - a^i_{ii}]}{\theta} - \frac{\Omega}{\theta}\frac{\partial r}{\partial t_i} \gtreqless 0, \\
\frac{d\hat{\alpha}^i}{d\tau_i} &= \frac{p_i q^i_i + p_j q^i_i + a^i_{ii} + a^i_{ji}}{\theta} - \frac{\Omega}{\theta}\frac{\partial r}{\partial \tau_i} > 0, \\
\frac{d\hat{\alpha}^i}{ds_i} &= -\frac{k^i_{Ri}}{\theta} - \frac{\Omega}{\theta}\frac{\partial r}{\partial s_i} < 0.
\end{aligned}
\tag{A.23}
$$

where $\Omega \equiv (k^i_{Rj} - k^i_{Ri}) + (k^i_{ij} - k^i_{ii}) + (k^i_{jj} - k^i_{ji})$.

Importantly, all effects working via a change in the interest rate disappear under symmetry. Moreover, the first term in the first line of (A.23) is also zero under symmetry. In addition, under symmetry, the effects of changes in the same policy variable are identical in absolute terms, and particularly $d\hat{\alpha}^i/d\tau_i = -d\hat{\alpha}^i/d\tau_j$ and $d\hat{\alpha}^i/ds_i = -d\hat{\alpha}^i/ds_j$.

Under symmetry, we also know from comparative statics and equation (A.10) that $dk^i_{Ri}/d\tau_i = -2f_q q_k (dk^i_{Ri}/ds_i)$, since $n = 2$ holds in this extension. Hence,

$$
\frac{d\hat{\alpha}^i}{ds_i}\frac{dk^i_{Ri}/d\tau_i}{dk^i_{Ri}/ds_i} = -\frac{2f_q q_s k^i_R}{\theta} \quad \Leftrightarrow \quad \frac{d\hat{\alpha}^i}{d\tau_i} - \frac{d\hat{\alpha}^i}{ds_i}\frac{dk^i_{Ri}/d\tau_i}{dk^i_{Ri}/ds_i} = \frac{2f_q\,(q^i_i - q_k k^i_R) + 2a^i}{\theta} > 0,
\tag{A.24}
$$

which will be used below.

With $n = 2$ countries, internationally mobile R&D units, and a continuum of heterogenous MNCs that are uniformly distributed on $\alpha \in [\underline{\alpha}, \overline{\alpha}]$, the optimized social welfare function (11) from the main model of non-cooperative decision making can be restated as

$$\begin{aligned}
\max_{t_i,\tau_i,s_i} W_i &= t_i \left\{\pi_N^i(RD^i) + (1-\hat{\alpha}^i+\hat{\alpha}^j)[f(k_{ii}^i,q_i^i) - f_q(k_{ii}^i,q_i^i)q_i^i - a_{ii}^i]\right. \\
&+ \left.(1-\hat{\alpha}^j+\hat{\alpha}^i)[f(k_{ij}^j,q_j^j) - f_q(k_{ij}^j,q_j^j)q_j^j - a_{ij}^j]\right\} \\
&+ \tau_i(1-\hat{\alpha}^i+\hat{\alpha}^j)\left\{f_q(k_{ii}^i,q_i^i)q_i^i + f_q(k_{ji}^i,q_i^i)q_i^i + a_{ii}^i + a_{ji}^i\right\} - (1-\hat{\alpha}^i+\hat{\alpha}^j)s_i k_{Ri}^i \\
&+ \gamma\left\{r\bar{k} + (1-t_i)\pi_N^i + (1-\hat{\alpha}^i)\Pi_{Mi}^i + \hat{\alpha}^i\Pi_{Mj}^i - \frac{\theta}{2}\hat{\alpha}^{i2}\right\}, \qquad \text{(A.25)}
\end{aligned}$$

where we have used $k_{Ri}^i = k_{Ri}^j$, $k_{ui}^i = k_{ui}^j$, and $a_{ui}^i = a_{ui}^j$. Furthermore, $(\theta/2)\hat{\alpha}^{i2} = \int_0^{\hat{\alpha}^i} \alpha_i \theta d\alpha^i$ gives total agency costs from placing the R&D unit abroad, summed over all MNCs headquartered in country i. Finally, $RD^i = (1-\hat{\alpha}^i+\hat{\alpha}^j)k_{Ri}^i$ captures total R&D spending in country i as driver for the spillover effect on national firms.

Optimal patent box tax rate. Differentiating national welfare W_i in (A.25) with respect to the corporate tax rate τ_i gives

$$\begin{aligned}
\frac{\partial W}{\partial \tau_i} &= [(1-\gamma)(1-\hat{\alpha}^i)+\hat{\alpha}^j][(p_i q_i^i + a_{ii}^i) + (p_j q_i^i + a_{ji}^i)] \\
&+ [t_i + \gamma(1-t_i)]\frac{\partial \pi_N^i}{\partial RD^i}\left\{\frac{\partial RD^i}{\partial \tau_i} - k_{Ri}^i\left(\frac{\partial \hat{\alpha}^i}{\partial \tau_i} - \frac{\partial \hat{\alpha}^j}{\partial \tau_i}\right)\right\} \\
&+ t_i(1-\hat{\alpha}^i+\hat{\alpha}^j)\left\{f_k\frac{\partial k_{ii}^i}{\partial \tau_i} - q_i^i\left[f_{qk}\frac{\partial k_{ii}^i}{\partial \tau_i} + f_{qq}q_k^i\frac{\partial k_{Ri}^i}{\partial \tau_i}\right] - \frac{\partial a_{ii}^i}{\partial \tau_i}\right\} \\
&+ t_i(1-\hat{\alpha}^j+\hat{\alpha}^i)\left\{f_k\frac{\partial k_{ij}^j}{\partial \tau_i} - q_j^i\left[f_{qk}\frac{\partial k_{ij}^j}{\partial \tau_i} + f_{qq}q_k^i\frac{\partial k_{Rj}^j}{\partial \tau_i}\right]\right\} \\
&+ \tau_i(1-\hat{\alpha}^i+\hat{\alpha}^j)\left\{q_i^i\left[f_{qk}\frac{\partial k_{ii}^i}{\partial \tau_i} + f_{qq}q_k^i\frac{\partial k_{Ri}^i}{\partial \tau_i}\right] + f_q q_k^i\frac{\partial k_{Ri}^i}{\partial \tau_i} + \frac{\partial a_{ii}^i}{\partial \tau_i}\right. \\
&+ \left. q_i^j\left[f_{qk}\frac{\partial k_{ji}^i}{\partial \tau_i} + f_{qq}q_k^j\frac{\partial k_{Ri}^i}{\partial \tau_i}\right] + f_q q_k^j\frac{\partial k_{Ri}^i}{\partial \tau_i} + \frac{\partial a_{ji}^i}{\partial \tau_i}\right\} - (1-\hat{\alpha}^i+\hat{\alpha}^j)s_i\frac{\partial k_{Ri}^i}{\partial \tau_i} \\
&+ \gamma\left\{\bar{k} - (k_i^i + k_j^i + k_{Ri}^i)\right\}\frac{\partial r}{\partial \tau_i} + \gamma\left[\Pi_{Mj}^i - \Pi_{Mi}^i - \hat{\alpha}^i\theta\right]\frac{\partial \hat{\alpha}^i}{\partial \tau_i} \\
&- \left\{t_i\left(\pi_{ii}^i - \pi_{ij}^i\right) + \tau_i\pi_R^i - s_i k_{Ri}^i\right\}\left(\frac{\partial \hat{\alpha}^i}{\partial \tau_i} - \frac{\partial \hat{\alpha}^j}{\partial \tau_i}\right) = 0. \qquad \text{(A.26)}
\end{aligned}$$

Using (9) and (27) and imposing symmetry, the first-order condition simplifies to

$$\begin{aligned}
\frac{\partial W}{\partial \tau_i} &= 2(1-\gamma)(f_q q^i + a_i) + [t_i + \gamma(1-t_i)]\frac{\partial \pi_N^i}{\partial RD^i}\left(\frac{\partial k_{Ri}}{\partial \tau_i} - 2k_{Ri}\frac{\partial \hat{\alpha}^i}{\partial \tau_i}\right) + (2\tau_i - t_i)\frac{\partial a_i}{\partial \tau_i} \\
&+ 2t_i f_k\frac{\partial k_i}{\partial \tau_i} + 2\tau_i f_q q_k^i\frac{\partial k_{Ri}}{\partial \tau_i} - s_i\frac{\partial k_{Ri}}{\partial \tau_i} + 2(\tau_i - t_i)q^i\left[f_{qk}\frac{\partial k_i}{\partial \tau_i} + f_{qq}q_k^i\frac{\partial k_{Ri}}{\partial \tau_i}\right] \\
&- 2[\tau_i\pi_R^i - s_i k_{Ri}]\frac{\partial \hat{\alpha}^i}{\partial \tau_i} = 0. \qquad \text{(A.27)}
\end{aligned}$$

Inserting the linear dependence of the royalty tax and the R&D subsidy from (A.10), the optimality condition turns into

$$\begin{aligned}\frac{\partial W}{\partial \tau_i} &= 2(1-\gamma)(f_q q^i + a_i) + (2\tau_i - t_i)\frac{\partial a_i}{\partial \tau_i} - 2 f_q q_k^i [t_i + \gamma(1-t_i)] \frac{\partial \pi_N^i}{\partial RD^i} \frac{\partial k_{Ri}}{\partial s_i} \\ &- 2 f_q q_k^i \left\{ 2 t_i f_k \frac{\partial k_i}{\partial s_i} + 2\tau_i f_q q_k^i \frac{\partial k_{Ri}}{\partial s_i} - s_i \frac{\partial k_{Ri}}{\partial s_i} + 2(\tau_i - t_i) q^i \left[f_{qk} \frac{\partial k_i}{\partial s_i} + f_{qq} q_k^i \frac{\partial k_{Ri}}{\partial s_i} \right] \right\} \\ &- 2[\tau_i \pi_R^i - s_i k_{Ri} - [t_i + \gamma(1-t_i)] \frac{\partial \pi_N^i}{\partial RD^i} 2 k_{Ri}] \frac{\partial \hat{\alpha}^i}{\partial \tau_i} = 0. \qquad \text{(A.28)}\end{aligned}$$

Optimal R&D subsidy. The first-order condition for the optimal R&D subsidy is

$$\begin{aligned}\frac{\partial W}{\partial s_i} &= -[(1-\gamma)(1-\hat{\alpha}^i) + \hat{\alpha}^j] k_{Ri}^i + [t_i + \gamma(1-t_i)] \frac{\partial \pi_N^i}{\partial RD^i} \left\{ \frac{\partial RD^i}{\partial s_i} - k_{Ri}^i \left(\frac{\partial \hat{\alpha}^i}{\partial s_i} - \frac{\partial \hat{\alpha}^j}{\partial s_i} \right) \right\} \\ &+ t_i (1 - \hat{\alpha}^i + \hat{\alpha}^j) \left\{ f_k \frac{\partial k_{ii}^i}{\partial d_i} - q_i^i \left[f_{qk} \frac{\partial k_{ii}^i}{\partial d_i} + f_{qq} q_k^i \frac{\partial k_{Ri}^i}{\partial s_i} \right] \right\} \\ &+ t_i (1 - \hat{\alpha}^j + \hat{\alpha}^i) \left\{ f_k \frac{\partial k_{ij}^j}{\partial s_i} - q_j^i \left[f_{qk} \frac{\partial k_{ij}^j}{\partial s_i} + f_{qq} q_k^i \frac{\partial k_{Rj}^j}{\partial s_i} \right] \right\} \\ &+ \tau_i (1 - \hat{\alpha}^i + \hat{\alpha}^j) \left\{ q_i^i \left[f_{qk} \frac{\partial k_{ii}^i}{\partial s_i} + f_{qq} q_k^i \frac{\partial k_{Ri}^i}{\partial s_i} \right] + f_q q_k^i \frac{\partial k_{Ri}^i}{\partial s_i} \right. \\ &\quad \left. + q_i^j \left[f_{qk} \frac{\partial k_{ji}^i}{\partial s_i} + f_{qq} q_k^j \frac{\partial k_{Ri}^i}{\partial s_i} \right] + f_q q_k^j \frac{\partial k_{Ri}^i}{\partial s_i} \right\} \\ &- (1 - \hat{\alpha}^i + \hat{\alpha}^j) s_i \frac{\partial k_{Ri}^i}{\partial s_i} + \gamma \left\{ \bar{k} - (k_i^i + k_j^i + k_{Ri}^i) \right\} \frac{\partial r}{\partial s_i} + \gamma \left[\Pi_{Mj}^i - \Pi_{Mi}^i - \hat{\alpha}^i \theta \right] \frac{\partial \hat{\alpha}^i}{\partial s_i} \\ &- \left\{ t_i \left(\pi_{ii}^i - \pi_{ij}^i \right) + \tau_i \pi_R^i - s_i k_{Ri}^i \right\} \left(\frac{\partial \hat{\alpha}^i}{\partial s_i} - \frac{\partial \hat{\alpha}^j}{\partial s_i} \right) = 0, \qquad \text{(A.29)}\end{aligned}$$

which simplifies by the usual steps under symmetry to eq.(29) in the main text.

Combining (A.28) and (29) and using (A.24) gives

$$f_q \; [q^i - q_k^i k_R^i] \left(1 - \gamma - 2\frac{B}{\theta} \right) + a_i \left(1 - \gamma - 2\frac{B}{\theta} \right) + \frac{2\tau_i - t_i}{2} \frac{\partial a_i}{\partial \tau_i} = 0, \qquad \text{(A.30)}$$

where $B = \tau_i \pi_R^i - s_i k_R^i + 2 k_R^i [t_i + \gamma(1-t_i)](\partial \pi_N^i / \partial k_R^i)$. Rearranging and using optimal profit shifting behavior $a_i^* = (t_i - \tau_i)/\beta$ gives (28) in the main text.

A.5 Location decision of the patent

Applying the Envelope theorem to (30) gives

$$
\begin{aligned}
\frac{\partial \Pi^i_{Mi}}{\partial t_i} &= -f(k^i_{ii}, q^i_i) + p_i q^i_i + a^i_{ii} - [k^i_{ii} + k^i_{ji} + k^i_{Ri}]\frac{\partial r}{\partial t_i},\\
\frac{\partial \Pi^i_{Mj}}{\partial t_i} &= -f(k^i_{ij}, q^i_j) + p_i q^i_j + a^i_{ij} - [k^i_{ij} + k^i_{jj} + k^i_{Rj}]\frac{\partial r}{\partial t_i},\\
\frac{\partial \Pi^i_{Mi}}{\partial \tau_i} &= -[p_i q^i_i + p_j q^i_i] - [a^i_{ii} + a^i_{ji}] - [k^i_{ii} + k^i_{ji} + k^i_{Ri}]\frac{\partial r}{\partial \tau_i},\\
\frac{\partial \Pi^i_{Mj}}{\partial \tau_i} &= -[k^i_{ij} + k^i_{jj} + k^i_{R_i}]\frac{\partial r}{\partial \tau_i},\\
\frac{\partial \Pi^i_{Mi}}{\partial s_i} &= k^i_{Ri} - [k^i_{ii} + k^i_{ji} + k^i_{Ri}]\frac{\partial r}{\partial s_i},\\
\frac{\partial \Pi^i_{Mj}}{\partial s_i} &= k^i_{Rj} - [k^i_{ij} + k^i_{jj} + k^i_{Rj}]\frac{\partial r}{\partial s_i}.
\end{aligned}
$$

The comparative-static effects of policy changes in country i on the optimal location of a patent are then obtained from differentiating eq. (31). This yields

$$
\begin{aligned}
\frac{d\hat{\alpha}^i_P}{dt_i} &= -\frac{[f(k^i_{ij}, q^i_j) - p_i q^i_j] - [f(k^i_{ii}, q^i_i) - p_i q^i_i] - [a^i_{ij} - a^i_{ii}]}{\theta_P} - \frac{\Omega}{\theta_P}\frac{\partial r}{\partial t_i},\\
\frac{d\hat{\alpha}^i_P}{d\tau_i} &= \frac{p_i q^i_i + p_j q^i_i + a^i_{ii} + a^i_{ji}}{\theta_P} - \frac{\Omega}{\theta_P}\frac{\partial r}{\partial \tau_i} > 0,\\
\frac{d\hat{\alpha}^i_P}{ds_i} &= \frac{k^i_{Rj} - k^i_{Ri}}{\theta_P} - \frac{\Omega}{\theta_P}\frac{\partial r}{\partial s_i},
\end{aligned}
$$

where Ω is the same as in (A.23). Imposing symmetry yields (32) in the main text.

The government's maximization problem under patent relocation is

$$
\begin{aligned}
\max_{t_i,\tau_i,s_i} W_i &= t_i \left\{\pi^i_N(RD^i) + (1-\hat{\alpha}^i_P)[f(k^i_{ii}, q^i_i) - f_q(k^i_{ii}, q^i_i)q^i_i - a^i_{ii}] + \hat{\alpha}^j_P[f(k^j_{ii}, q^j_i) - f_q(k^j_{ii}, q^j_i)q^j_i - a^j_{ii}]\right.\\
&+ \left.(1-\hat{\alpha}^j_P)[f(k^j_{ij}, q^j_j) - f_q(k^j_{ij}, q^j_j)q^j_j - a^j_{ij}] + \hat{\alpha}^i[f(k^i_{ij}, q^i_j) - f_q(k^i_{ij}, q^i_j)q^i_j - a^i_{ij}]\right\}\\
&+ \tau_i \left\{(1-\hat{\alpha}^i_P)\left[f_q(k^i_{ii}, q^i_i)q^i_i + f_q(k^i_{ji}, q^i_i)q^i_i + a^i_{ii} + a^i_{ji}\right]\right.\\
&+ \left.\hat{\alpha}^j_P\left[f_q(k^j_{ii}, q^j_i)q^j_i + f_q(k^j_{ji}, q^j_i)q^j_i + a^j_{ii} + a^j_{ji}\right]\right\} - (1-\hat{\alpha}^i_P)s_i k^i_{Ri} + \hat{\alpha}^i_P s_i k^i_{Rj}\\
&+ \gamma\left\{r\bar{k} + (1-t_i)\pi^i_N(RD^i) + (1-\hat{\alpha}^i_P)\Pi^i_{Mi} + \hat{\alpha}^i_P\Pi^i_{Mj} - \frac{\theta}{2}\hat{\alpha}^{i2}_P\right\}, \qquad \text{(A.31)}
\end{aligned}
$$

where now $RD^i = (1-\hat{\alpha}^i_P)k^i_{Ri} + \hat{\alpha}^i_P k^i_{Rj}$.

Differentiating (A.31) with respect to the tax rate τ_i, using the market equilbria (9) and (32) and imposing symmetry, gives the first-order condition

$$\begin{aligned}\frac{\partial W}{\partial \tau_i} &= 2(1-\gamma)(f_q q^i + a_i) + [t_i + \gamma(1-t_i)]\frac{\partial \pi_N^i}{\partial RD^i}\frac{\partial k_R^i}{\partial \tau_i} + (2\tau_i - t_i)\frac{\partial a_i}{\partial \tau_i} \\ &+ 2t_i f_k \frac{\partial k_i}{\partial \tau_i} + 2\tau_i f_q q_k^i \frac{\partial k_{Ri}}{\partial \tau_i} - s_i \frac{\partial k_{Ri}}{\partial \tau_i} + 2(\tau_i - t_i)q^i \left[f_{qk}\frac{\partial k_i}{\partial \tau_i} + f_{qq} q_k^i \frac{\partial k_R^i}{\partial \tau_i}\right] \\ &- 2\tau_i \pi_R^i \frac{\partial \hat{\alpha}_P^i}{\partial \tau_i} = 0. \end{aligned} \tag{A.32}$$

Inserting the linear dependence relations in (A.10), the optimality condition turns into

$$\begin{aligned}\frac{\partial W}{\partial \tau_i} &= 2(1-\gamma)(f_q q^i + a_i) + (2\tau_i - t_i)\frac{\partial a_i}{\partial \tau_i} - 2f_q q_k^i [t_i + \gamma(1-t_i)]\frac{\partial \pi_N^i}{\partial RD^i}\frac{\partial k_R^i}{\partial s_i} \\ &- 2f_q q_k^i \left\{2t_i f_k \frac{\partial k_i}{\partial s_i} + 2\tau_i f_q q_k^i \frac{\partial k_R^i}{\partial s_i} - s_i \frac{\partial k_R^i}{\partial s_i} + 2(\tau_i - t_i)q^i \left[f_{qk}\frac{\partial k_i}{\partial s_i} + f_{qq} q_k^i \frac{\partial k_R^i}{\partial s_i}\right]\right\} \\ &- 2\tau_i \pi_R^i \frac{\partial \hat{\alpha}_P^i}{\partial \tau_i} = 0. \end{aligned} \tag{A.33}$$

The first-order condition for the optimal R&D subsidy s_i is derived analogously and leads to eq. (34) in the main text. Combining (A.33) with (34) and using (A.31) leads to

$$(1-\gamma)f_q\ [q^i - q_k^i k_R^i] + (1-\gamma)a_i + \frac{2\tau_i - t_i}{2}\frac{\partial a_i}{\partial \tau_i} - 2\frac{B_P}{\theta_P}[f_q q^i + a_i] = 0, \tag{A.34}$$

where $B_P = \tau_i \pi_R^i$. Using $a_i^* = (t_i - \tau_i)/\beta$ from optimal profit shifting behavior gives

$$\frac{t_i}{\tau_i} = \frac{4 - 2\gamma - 4B_P/\theta_P}{3 - 2\gamma - 4B_P/\theta_P} - \frac{\beta f_q}{\tau_i}\frac{(2 - 2\gamma - 4B_P/\theta_P)q^i - (2-2\gamma)q_k k_R^i}{(3 - 2\gamma - 4B_P/\theta_P)} \tag{A.35}$$

Decomposing the second term in (A.35) gives eq. (33) in the main text.

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