Technology and Tax Systems

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Abstract

Technological innovations facilitating e-commerce have had major effects on consumer behavior and firm organization in the retail sector, but the effects of these new transaction technologies on fiscal systems remain unknown. We extend models of commodity tax competition to include multiple types of commodities, trade, and remote commerce, assuming, in accordance with current policy, that e-commerce is taxed at destination while cross-border shopping is taxed at origin. When the cost of online shopping falls, we show that equilibrium tax rates and revenues decrease in large, core jurisdictions but increase in small, peripheral ones, reducing tax differentials. Policy commentators warn that e-commerce erodes tax revenue – true enough for some governments – but, more accurately, changing transaction costs can generate entirely new commercial and fiscal equilibria that ultimately "redistribute" tax revenues from jurisdictions with concentrations of traditional vendors toward others. With some reinterpretation, the model is also adapted to analyze profit-tax competition when firms can respond to high taxes both through profit-shifting and through relocation, each dependent on transactions costs. Changes in technology may again redistribute tax revenues from high-tax to low-tax jurisdictions.

Keywords: Commodity Tax, Retail Shopping, Agglomeration, e-Commerce, Enforcement, Fiscal Competition, Corporate Taxes.

JEL classifications: H25, H71, H73, L81, R50

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1 Introduction

Technological innovations that reduce transportation and communication costs arguably intensify fiscal competition among governments and constrain their ability to enforce taxes. Internet commerce has dramatically expanded the geographical scope of fiscal competition as consumers and businesses can interact "virtually" on a global scale. At the same time, it poses new challenges for tax administration; indeed, at the dawn of the internet era, e-commerce appears to have facilitated tax avoidance (Goolsbee 2000; Einav et al. 2014). Thus, much discussion has focused on the idea that technological innovation in general, and e-commerce in particular, may limit the taxing powers of governments, putting downward pressure on tax rates and revenues. Less appreciated is the possibility that technological change, together with suitable policy and institutional adjustments, may open up opportunities for governments to administer and enforce taxes in new ways, perhaps offsetting or even reversing downward pressure on tax rates and revenues arising from increased mobility.

More specifically, technological improvements affect fiscal policy in multiple ways. First, they reduce transaction costs that increase mobility of the base (globalization, transportation networks). Second, they may shift some of the tax base to more easily monitored transactions (computer auditing, electronic reporting). A priori, there is no reason to expect that these changes should affect all jurisdictions in the same way, especially in the presence of asymmetries in jurisdictional size, resource endowments, and agglomeration effects. We explore these two mechanisms within the context of a model of consumption tax competition that explicitly takes into account the implications of e-commerce. In this model, technological shocks resulting from declining costs of internet commerce allow consumers – who previously could only buy goods from nearby jurisdictions – to buy goods from vendors "worldwide". However, given recent institutional reforms, governments – who previously could only tax sales within their boundaries – can now monitor and tax purchases made by their residents from remote vendors. In the presence of initially asymmetrically located shopping or trade opportunities, these effects operate differently on different jurisdictions. The first of these mechanisms limits the revenue raising capacity of jurisdictions where consumption purchases are initially concentrated, while the second mechanism expands the taxing capacity of others.

Our model starts from the special case captured in classic studies such as Kanbur and Keen (1993) and Nielsen (2001) in which tax competition arises because households can engage in cross-border shopping and in which a single homogeneous commodity is taxed on an origin basis (i.e., taxes are levied where vendors are located).[1] In this class of models,

[1] Other studies include Mintz and Tulkens (1986), Braid (1993), Lockwood (1993), Trandel (1994), Haufler

all jurisdictions – both large and small – lower their tax rates as the cost of cross-border shopping falls. We extend this framework by developing a more general model of cross-border commerce that reflects interjurisdictional trade arising from local or regional specializations in the availability of heterogeneous commodities. Cross-border shopping remains taxed on an origin basis, but e-commerce is assumed to be taxed on a destination basis (i.e., taxes are levied where consumers reside). In this model, the relative cost of cross-border shopping and e-commerce is critical. In the prior literature, falling transaction costs reduce tax rates for all jurisdictions, but we find instead that a decline in the cost of e-commerce affects competing governments in different directions. Tax rates and revenues fall only in large jurisdictions, rising instead in small jurisdictions. The latter of these effects is a result of the internet shifting some transactions to a more readily monitored tax base, which benefits jurisdictions with initially less taxable activity. We derive these results initially in a model grounded in the spatial framework used in previous studies. We later show that the results can be generalized by allowing for many commodity types with complex trade patterns, different transaction costs for different commodities, imperfect enforcement, and differentiated tax treatment of commodities. This allows for many more applications than might be initially apparent, such as excise taxation and taxation of interregional and international trade.

Furthermore, the insights of our model are not limited to consumption taxes. As a second illustration of our general theme, a later section adapts the model to study a very different policy issue, the competition among governments that impose taxes on business incomes. The model recognizes that firms may avoid high taxes by either (or both) of two means: first, by using costly organizational forms, accounting procedures, or other methods to shift income to jurisdictions that offer more favorable tax treatment, or second, by relocating productive business activities, at a cost, to lower-tax jurisdictions. In this context, technological change is reflected in the costs of each type of tax avoidance; in either case, technological changes that lower these costs would tend, plausibly, to intensify fiscal competition. As in the consumption-tax case, however, we find that matters are not so simple. A decline in the cost of profit shifting intensifies tax competition and lowers tax rates in all jurisdictions. However, reductions in the cost of (real) business relocation may increase equilibrium tax rates and tax revenues for some low-tax jurisdictions, even as they put downward pressure on the taxes of high-tax jurisdictions. As in the consumption tax case, these divergent results arise because technological change alters the spatial distribution of the portion of the tax base that can easily be monitored.

(1996), Lockwood (2001), Nielsen (2002), Devereux et al. (2007), Braid (2013) and Agrawal (2015).

2 Cross-border Shopping, Internet Commerce, and Taxation

As indicated, previous research on strategic consumption taxation has focused primarily on simple models with a single taxed commodity which households may purchase either in their home jurisdictions or, at a cost, by traveling to a neighboring lower-tax jurisdiction. The ability of households to take advantage of lower tax rates in this manner presupposes that their purchases are taxed on an origin basis, a reasonable supposition when there are no effective fiscal frontiers and when governments have no feasible methods to monitor and tax the extra-jurisdictional purchases of their residents. Although it can certainly arise at national or state/provincial boundaries, such origin-based taxation is especially important at the local level, where households may cross jurisdictional boundaries to buy goods and services at major shopping centers or, for commuters, near their places of employment.

Such cross-border shopping, however, constitutes only one portion of total consumption. At the national level, most consumption spending is taxed on a destination basis because households buy goods primarily in their country of residence. Furthermore, as discussed below, fiscal systems are moving toward destination-based taxation of e-commerce. To the extent that e-commerce displaces cross-border purchases that would otherwise occur in neighboring jurisdictions, where they would be taxed on an origin basis, it limits the tax arbitrage opportunities from cross-border shopping.

These fundamental features of the consumption tax landscape cannot be represented adequately in a model with a single homogeneous commodity, taxed only on an origin basis. Such a model can capture neither the pervasive importance of interjurisdictional trade flows (whether at the level of localities, states/provinces, or nations) nor the co-existence of both origin and destination components in most observed tax systems. Furthermore, whereas cross-border shopping by households (the focus of previous studies) is important for some types of commodities, it is far less so for others; likewise, trade via e-commerce is highly important for some types of commodities, but far less so for others.[2] These considerations highlight the importance of product heterogeneity.

Classically, cross-border commerce can arise from asymmetries in the intrinsic technological or resource endowment attributes of jurisdictions (so-called "first nature" or "fundamentals"). In addition, it is now well-recognized (Head and Mayer 2004; Ottaviano and Thisse 2004; Rosenthal and Strange 2004) that economies of scale alone can create asymmetric agglomerations in which firms specialize in the production of particular commodities (so-called

[2] Overall, e-commerce represents 10% of retail sales. See Goolsbee (2000), Ellison and Ellison (2009), Goolsbee et al. (2010), and Einav et al. (2014) for the effect of tax rates on e-commerce.

"second nature"). As shown in the new economic geography literature (Kind et al. 2000; Baldwin and Krugman 2004; Borck and Pflüger 2006), agglomerations can interact with trade costs in ways that affect not only trade patterns, but also fiscal systems.[3]

For whatever reason trade exists, cross-border commerce reflects underlying asymmetries in production patterns and product availability, which, accordingly, are key elements of our model. Depending on the relative cost of transportation to brick-and-mortar retailers and online commerce as well as interjurisdictional tax differentials, households purchase these commodities either by cross-border shopping in a neighboring jurisdiction or through internet purchases. The former are assumed to be taxed on an origin basis whereas the latter are taxed on a destination basis, in accordance with a recent Supreme Court decision in the U.S. and a European Commission directive in the E.U.[4] Thus, online purchases provide tax authorities with a "technology" that enhances the enforcement of taxes at destination.

With these considerations in mind, we develop a theoretical model that simultaneously accommodates the existence of production asymmetries that attract cross-border shoppers as well as including e-commerce, while still retaining key elements of the simple spatial competition framework that underlies most previous research on consumption-tax competition. We postulate a baseline model with just two jurisdictions in sufficiently close proximity that households can shop in the neighboring jurisdiction at modest cost. Whereas previous studies allow for only one type of (non-numéraire) commodity that is produced and sold in both jurisdictions (a "standard" commodity), our model postulates that some types of "specialized" commodities can only be purchased in specific "agglomeration points." Consistent with observed practices, origin-based taxation for cross-border shoppers is assumed. For other types of transactions, such as commodities that are purchased by consumers in their place of residence (whether produced there or imported from elsewhere) or remote

[3]These studies analyze capital tax competition. A decline in trade costs may result in tax rates converging in some cases and diverging in others. In contrast to these studies, which focus on trade costs and the endogenous development of agglomerations, we simply treat agglomerations as one of several possible sources of asymmetric spatial distribution of economic activity. Unlike them, we focus on commodity taxes and consider how technology affects the spatial distribution of commerce and tax enforcement, particularly through trade with "the rest of the world" via e-commerce.

[4]In the U.S., retail sales are subject to state and local taxes. A major recent decision by the U.S. Supreme Court in the case of *South Dakota v. Wayfair, Inc.*, 585 U.S. __ (2018), allows states to require firms to remit state and local taxes if a remote vendor has sufficient "economic and virtual contacts" in a state, a standard that overturns the prior physical presence standard under *Quill Corp. v. North Dakota*, 504 U.S. 298 (1992). This will effectively shift taxes on online purchases to a destination basis, given that many vendors satisfy this standard. In the E.U. as well, remote sales are generally taxed at the place of consumption under a distance selling rule where suppliers with a sufficiently large volume of sales to a member state are required to remit taxes to the country where consumption takes places (Art. 34 Council Directive 2006/112/EC). For additional details, see Agrawal and Fox (2017).

purchases, jurisdictions are assumed to impose taxes on a destination basis. The model incorporates "transactions costs" that reflect transportation, communication, distance, and technological factors that determine the relative proportions of cross-border shopping and remote commerce (and, thus, the balance between origin-based and destination-based taxation). A decline of the transaction cost of buying online has two key effects: (a) it increases the mobility of the tax base by making shoppers more footloose as consumers have increased access to vendors "worldwide" and (b) in view of the above-noted institutional adaptations, it facilitates the enforcement of destination-based taxes on remote purchases. These two mechanisms interact with the initial distribution of economic activity and agglomerations to produce asymmetric effects on fiscal systems.

By allowing for different constellations of relative transactions costs, we analyze commodity tax competition in (i) a world where remote commerce is prohibitively costly (a representation of a world with no internet technology, similar to previous research), (ii) a world in which remote commerce occurs, but not to the exclusion of other, more traditional transactions patterns, and (iii) a world, perhaps to materialize in the future, where remote commerce is nearly costless. The second case, which corresponds to current circumstances and which exhibits the full range of interactions between technologies and fiscal systems, is the main focus of attention.

As in more standard models (Kanbur and Keen 1993; Nielsen 2001), jurisdiction size continues to shape the strategic interactions between competing jurisdictions and their equilibrium tax policies. However, many other factors now also come into play. The analysis shows that agglomerations, the importance to consumers of specialized commodities relative to traditional standardized goods, and access to remote vendors all affect equilibrium tax rates, tax revenues, and cross-border commerce, including both trade and cross-border shopping. Indeed, although tax considerations certainly do affect cross-border shopping, we show that there can easily be two-way cross-border transactions between neighboring jurisdictions, a realistic equilibrium property that cannot occur in standard models where jurisdictions differ only in size and consumers buy homogeneous commodities. This is even more the case when the model is extended to allow for jurisdictions to contain agglomerations that support multiple types of production, one of several generalizations that we explore.

Some of the effects of technological change are intuitively plausible, while others are more surprising. For instance, reductions in the cost of internet access *decreases* the tax rates and tax revenues of large cities (those that are net exporters of specialized commodities to neighboring jurisdictions) but result in *increases* in the tax rates and revenues of small jurisdictions. Although tax revenues for each jurisdiction move in opposite directions, the combined tax revenues of large and small jurisdictions *decrease* as the cost of e-commerce declines. These results arise because variations in the relative cost of e-commerce change how and where transactions occur and are taxed, reducing the tax bases of large jurisdictions while increasing both the tax bases and the tax-enforcement capabilities of small jurisdictions.

3 Model of Cross-border and Remote Commerce

Our goal is to develop a general framework for the analysis of strategic sales / consumption / commodity tax interactions that can encompass a range of different transactions technologies. It builds upon the well-known Hotelling linear spatial model and, in particular, upon previous contributions by Kanbur and Keen (1993), Nielsen (2001), and others who adapt the Hotelling model to tax competition problems. In our model, critical transactions-cost parameters capture the state of transactions technology (specifically, transportation costs and internet access costs), determining the environment within which governments and consumers make their policy and consumption choices. These simultaneous interactions endogenously determine the equilibrium configuration of transactions patterns (where and how transactions occur) and of public policies. As will become clear, there are extreme or polar parameter constellations that produce equilibria with minimal (zero) online commerce or with maximal online commerce. Of greatest interest, however, are parameters that allow for the coexistence of traditional "brick and mortar" and online shopping opportunities, such as is presently observed; we treat this type of equilibrium as our central case.

A principal objective of the analysis is to derive (local and global) comparative statics results, showing how changes in technological parameters (most notably, reduced cost and greater ease of internet commerce) affect all of the key endogenous variables in the model: the volume of cross-border shopping, internet shopping, tax rates, and tax revenues. One particular application of the model is to local government taxation in the U.S., where retail shopping is concentrated in larger localities and municipalities levy sales taxes. For the purposes of this application, as well as for comparability with previous studies, it is helpful to interpret the model as one with a "central point of agglomeration" where, in addition to *standardized* or "nonspecialized" retail goods and services (routine consumption items, often relatively perishable and frequently purchased), households can obtain more *specialized* commodities (those generally not available in more peripheral locations). We assume that "standardized" commodities, as a group, are typically too difficult to deliver or to market via internet commerce. By contrast, we assume that the more specialized commodities, available at the agglomeration point (initially only one, relaxed subsequently), can also be purchased from remote vendors through online transactions, to a degree that depends on the technology of internet marketing, purchasing mechanisms, packaging, and delivery.[5] These two categories of commodities, standardized and specialized, trade at fixed relative producer prices, all normalized to one. This assumption is consistent with Kanbur and Keen (1993) and Nielsen (2001).[6]

To capture these ideas, begin with the line segment $[0, H]$ along which immobile individuals are situated with unit density. Each household has some exogenously fixed endowment of a numéraire commodity which it uses to purchase one unit of the standardized commodity and S units of the specialized commodity.[7] The segment is exogenously divided at point $h \geq H/2$, which demarcates the (large) central jurisdiction (the "city") from the surrounding (small) area (the "suburb" or "hinterland"), with 0 serving as the (initially only) point of agglomeration at which the specialized commodity may be purchased. Our model is a simplification of a more general model, where the small jurisdiction also contains a point of agglomeration at which a different set of specialized products may exist. Such a model, with bidirectional flows of the specialized good, may be more appropriate at the international level but raises additional technical issues. For this reason, we later explain how the model can be generalized to multiple points of agglomeration and explain how this affects our results.

The standardized good is available at every location in this line segment; households may thus purchase it with no transportation costs. Alternatively, a household located at some point $x \in [0, H]$ may nonetheless elect to travel to the city/suburban boundary in order to purchase the standardized commodity in the other jurisdiction. This entails a transportation

[5] A sharp distinction between these two types of goods is somewhat overdrawn; on the other hand, the fact that some goods and services are much more amenable to online commerce than others is unmistakable and is reflected in observed transactions arrangements. Nevertheless, this two-fold categorization of commodities, which simplifies the presentation of the model, is not essential to the main results. Later we discuss how the results generalize to multiple traded commodities, purchased in varying quantities, at varying transactions costs, and originating in multiple jurisdictions. If the budget share of specialized goods becomes large relative to standardized goods, then having multiple specialized commodities with heterogeneous transaction costs makes the distinction between these goods inconsequential. We discuss these issues in section 6.

[6] As in previous literature, prices are expressed relative to a numéraire. We omit detailed treatment of economies of scale, product differentiation, or firm behavior, although the production side of the model is consistent with many models of agglomerated and dispersed production such as those used in the literature of urban economics and international economics.

[7] As in most analyses of of spatial competition, we assume that the demands for taxed commodities are both price-inelastic and income-inelastic in the neighborhood of an equilibrium. While this assumption is maintained for technical reasons, the results of the model certainly extend to more general cases. In particular, the results would remain unchanged with non-zero income and price elasticities of demand, provided that these are sufficiently small in the neighborhood of an equilibrium.

cost of $d > 0$ per unit of distance and thus a total transportation cost of $d|x-h|$, an avoidable cost that at least some households will nonetheless make if justified by tax differentials.

A consumer located at point x must incur a transportation cost of $D > 0$ per unit of distance, and thus a total transportation cost of Dx, to purchase the specialized good at the agglomeration point 0. If remote (internet, mail order) access and transactions are attractive, consumers may escape this transportation cost, incurring instead a fixed cost of E for all remote purchases. These might include shipping costs, which are independent of distance, internet access charges, and any added costs of buying online. Then, E/D represents the relative cost of remote commerce and brick-and-mortar shopping per unit distance.

As a matter of interpretation, "transportation" or "transaction" costs refer to all costs incurred, including direct out-of-pocket travel costs and the value of travel time on the part of consumers, the costs of delivery (whether incurred directly by consumers or by vendors[8]), information, marketing, and communication costs, transaction settlement costs (credit card fees, account management, etc.), costs of internet services, and others. The costs of internet transactions in particular are rapidly evolving due to new methods of organizing transactions, sometimes involving new intermediaries, new mechanisms for product display, new order fulfillment mechanisms, and, no doubt, other innovations are still on (or beyond) the horizon. This simple three-parameter (d, D, E) specification[9] suffices to capture the most important considerations, namely, changes in the *relative costs* of various transactions arrangements, especially the costs of internet relative to traditional transactions technologies.

Both jurisdictions are assumed to impose *origin*-type consumption taxes on both types of goods when transacted through *traditional* means, reflecting two institutional considerations: first, tax authorities may not have the legal power to impose taxes on non-resident households (i.e., cross-border shoppers), and second, taxes imposed on vendors at the point of sale are more easily administered than taxes imposed on purchasers due to the relative ease with which authorities can identify, audit, and (if need be) punish non-compliance. By contrast, we assume that each government taxes online purchases made by their residents so that this portion of the sales tax is implemented on a *destination* basis. This is consistent with current

[8]The costs incurred by vendors – say, the delivery of an appliance to a residence – may be directly paid by vendors. However, vendors must recoup these costs, explicitly or implicitly, and they may be treated "as if" paid directly by consumers in the form of equivalent price markups. This simplification neglects some second-order questions about whether or not taxes are paid on these transactions costs.

[9]One could view our model as having four parameters, one of which is the cost of purchasing standardized commodities remotely. This parameter is suppressed because we assume it is prohibitively costly due to the characteristics of these goods as discussed above. Of course, future technological changes may make online purchases of these products more feasible.

practices, both at the local and national levels given that recent reforms allow states and countries to require remote vendors to remit taxes if they have sufficient economic presence in the jurisdiction. Given the normalization of producer prices, let T and t, respectively, denote the rate at which sales to consumers are taxed in the city and in the suburb.

Before proceeding to the formal analysis, note that the linear spatial structure, with an agglomeration point at 0 and with households distributed linearly over the unit interval, also captures the case of a symmetric two-sided spatial structure with a central city and an suburb on either side, merely by reflection around 0. Furthermore, it captures the case of a central city surrounded by any number of identical suburbs situated around it, so long as all travel to the agglomeration point is radial. The presentation is limited to the simple two-jurisdiction framework, however, for notational convenience.

Quite aside from the explicit formulation of spatial competition by neighboring localities in a linear model, other interpretations of the model are possible; indeed, many studies have used this basic linear model to investigate international tax competition. Following but generalizing previous studies, our competing jurisdictions could be countries whose consumers purchase standardized commodities by cross-border shopping. However, the consumers also purchase specialized products only available outside their country of residence. In this interpretation, as discussed in section 6, "agglomerations" represent country-specific sources of specialized commodities, whether arising from "first" or "second" nature. Another interpretation is offered by Keen and Konrad (2013), who explain how the Kanbur and Keen (1993) model can be utilized to describe profit-shifting behavior in a system of business profit taxation. Section 7 shows how the present model, with its two categories of transactions, can simultaneously accommodate both profit-shifting and real business investment relocation.

4 Equilibria with Revenue-maximizing Governments

The revenues obtained by each government depend both on its tax rate and on the total size of its tax base; the latter consists of taxable sales of both standardized and specialized commodities. Because consumers respond to taxes in choosing where to make their purchases, each jurisdiction's tax bases depend on its own tax rate and on the other jurisdiction's tax rate. Letting upper- and lower-case letters designate the large and small jurisdictions, and letting S and N designate the specialized and standardized (nonspecialized) commodities, respectively, the two bases for the large and small jurisdictions may be written as $B_i(T,t)$ and $b_i(t,T)$, for $i = N, S$. The revenues of each jurisdiction are

$$\begin{aligned} R(T,t) &= T(B_N(T,t) + B_S(T,t)) \\ r(t,T) &= t(b_N(t,T) + b_S(t,T)), \end{aligned} \tag{1}$$

following the convention of writing the own-tax rates first in each function.

Both for the sake of tractability and for comparability with previous literature, we suppose that governments choose their taxes non-cooperatively in order to maximize revenues.[10] Thus, a Nash non-cooperative equilibrium is a pair (T_*, t_*) satisfying the usual conditions:

$$\begin{aligned} T_* = &\quad argmax_{<T>} R(T, t_*) \\ t_* = &\quad argmax_{<t>} r(t, T_*). \end{aligned} \tag{2}$$

Even without exploiting any special assumptions about transactions technologies, or the spatial structure, one may use the first-order conditions for revenue maximization to characterize the Nash equilibrium tax rates for each jurisdiction. In each case, the choice of tax rate entails a tradeoff between the extra revenue generated by a higher tax rate and the shrinkage of the tax bases. Let ϵ_{B_i}, $i = N, S$, denote the (absolute value of the) elasticity of each of the two bases for the large jurisdiction with respect to $1 + T$, with ϵ_{b_i} defined analogously for the small jurisdiction.[11] The equilibrium tax rates must satisfy

$$\begin{aligned} \frac{T}{1+T} &= \frac{1}{\Omega\epsilon_{B_N} + (1-\Omega)\epsilon_{B_S}} \\ \frac{t}{1+t} &= \frac{1}{\omega\epsilon_{b_N} + (1-\omega)\epsilon_{b_S}} \end{aligned} . \tag{3}$$

where $\Omega = B_N/B$ is the fraction of the large jurisdiction's total tax base, $B = B_N + B_S$, that is accounted for by the standardized commodity and $\omega = b_N/b$ is the corresponding share of the small jurisdiction's total tax base, $b = b_N + b_S$.

These conditions are variants of familiar inverse-elasticity formulae and they indicate the general nature of the solutions to the revenue-maximization problems. These weighted-average formulae can easily be generalized further to allow for commodity-specific elasticities and consumption shares in a model with an arbitrary number of commodities in which the

[10]The focus of analysis in the present paper is not on the objectives that governments may pursue but rather on the constraints, as determined by technology and transactions costs, that shape the environment within which they choose their policies. Whether or not the specification of different government objectives would have any qualitative effect on policy responses to changes in objectives remains to be investigated.

[11]To be precise, $\epsilon_{B_i} = \frac{1+T}{B_i}\frac{\partial B_i(T,t)}{\partial T}$, which depends on (T,t). For a local characterization of a Nash equilibrium, these elasticities are evaluated at (T_*, t_*), ignoring discontinuities at corners for the moment.

relative ease of traditional and remote transactions may be product-specific.

As is typical of inverse-elasticity formulations of tax rules, however, the expressions in (3) are highly implicit, and for this reason, they do not lend themselves to clear-cut comparative statics results. To see this, let us simplify the problem, as in the analysis to follow below, by assuming that individual household demands for all commodities are perfectly price-inelastic. Under this assumption, the tax bases of each jurisdiction can only vary because of differences in where and how commodities are purchased. Suppose, then, that we wish (for instance) to ascertain how a decline in the relative cost of remote commerce and brick-and-mortar shopping E/D affects equilibrium tax rates, as implicitly determined by (3). If tax rates were unchanged, fewer households in the small jurisdiction would choose to purchase the specialized commodity in the city, so that B_S would fall and b_S would rise. At unchanged tax rates, a decrease in B_S would raise the elasticity of the specialized base (ϵ_{B_S}) for the city, suggesting that the revenue-maximizing tax rate for the large jurisdiction must fall, and conversely for the small jurisdiction. However, because $1 - \Omega$ falls as B_S falls, and because $1 - \omega$ rises as b_S rises, the weights attached to each of the product-specific elasticities in (3) must change, and the relative sizes of these elasticities cannot be determined *a priori*. Thus, there can be no presumption, in general, that the equilibrium tax rate T must rise, or that t must fall, as the costs of online commerce fall, even abstracting from the confounding effects of price-sensitive individual demand functions.

In order to obtain more definite results, let us utilize the linear transactions cost assumptions introduced in Section 3 in order to specify the exact forms of the revenue functions for both jurisdictions, which are determined by the size of each of the two bases. It is natural to assume that consumers purchase each commodity in the least-cost fashion, taking both taxes and transactions costs into account. For the standardized commodity, this determines a critical location x_N^* defined by

$$x_N^* = h + \frac{t - T}{d} \tag{4}$$

such that all consumers located distances $x \leq x_N^*$ from the city center purchase the standardized commodity in the city, whereas all others purchase in the suburb. This cutoff rule, which depends on the tax rate differential and on the technological parameter d, determines the standardized commodity tax base for each jurisdiction.

For the specialized commodity, although residents in the city may purchase either at the center or online, due to destination taxation of online sales, their total consumption of this commodity is taxed in either case at the city rate of T, an invariant contribution (of hS) to the city tax base. On the other hand, suburban residents purchase the specialized commodity either at the center where they pay a tax of T and add to the city tax base, or online, paying a tax of t and adding to the suburban tax base. The critical location is defined by

$$x_S^* = \frac{(t-T)S}{D} + \frac{E}{D} \tag{5}$$

such that any (suburban) residents situated at locations $x \in (h, H]$ for whom $x \geq x_S^*$ find it preferable to purchase the specialized commodity online rather than at the city center. Note that this cutoff rule depends on the ease of remote purchases relative to traditional commerce, E/D, as well as the tax rate differential. Figure 1 shows the geography of the model and possible locations of the cutoff rules when $T > t$ and some small jurisdiction residents buy online.

Taking into account the fact that all city households reside at some point $x \in [0, h]$ and that all suburban residents reside at a point $x \in (h, H]$, and taking into account the administrative and institutional factors described above, the tax bases for each jurisdiction may now be written formally. Taking first the standardized commodity,

$$B_N(T,t) = \begin{cases} H & \text{if } x_N^* \geq H & \text{case: ``all city''} \\ h + \frac{t-T}{d} & \text{if } x_N^* \in [0,\ H] & \text{case: ``interior''} \\ 0 & \text{if } x_N^* \leq 0 & \text{case: ``all suburb''} \end{cases} \tag{6}$$

for the large jurisdiction and, because $B_N + b_N = H$,

$$b_N(t,T) = \begin{cases} 0 & \text{if } x_N^* \geq H & \text{case: ``all city''} \\ H - h + \frac{T-t}{d} & \text{if } x_N^* \in [0,\ H] & \text{case: ``interior''} \\ H & \text{if } x_N^* \leq 0 & \text{case: ``all suburb''} \end{cases} \tag{7}$$

for the small jurisdiction. For the specialized commodity, the corresponding definitions are

$$B_S(T,t) = \begin{cases} SH & \text{if } x_S^* \geq H & \text{case: ``all city''} \\ S\left(\frac{E+(t-T)S}{D}\right) & \text{if } x_S^* \in [h,\ H] & \text{case: ``some online''} \\ Sh & \text{if } x_S^* \leq h & \text{case: ``maximal online''} \end{cases} \tag{8}$$

and

$$b_S(t,T) = \begin{cases} 0 & \text{if } x_S^* \geq H & \text{case: ``all city''} \\ SH - S\left(\frac{E+(t-T)S}{D}\right) & \text{if } x_S^* \in [h,\, H] & \text{case: ``some online''} \\ S(H-h) & \text{if } x_S^* \leq h. & \text{case: ``maximal online''} \end{cases} \tag{9}$$

for the large and small jurisdictions, respectively. Critically, because taxes on e-commerce are sourced at destination, residents of the large jurisdiction always pay its tax rate on specialized purchases regardless of the model of commerce; however, residents of the small jurisdiction may pay taxes to the large or small jurisdiction depending on if the specialized good is purchased online or not.

The size of each base depends on the tax rate *differential*, the bases are continuous but *not* differentiable in the technological parameters and tax differentials, and there are corner conditions (no base can be negative, no base can exceed the maximum). Substituting these expressions for the tax bases into (1), it is apparent that the revenue functions are piecewise continuous – but not continuously differentiable – functions of the tax rates. Figure 2 shows an example of the revenue functions, given a parameter constellation for which an equilibrium exists in the "some online" case.

Given the definitions of the equilibrium tax rates and the tax bases presented in (6)–(9), it is evident that the structure of commercial activity (i.e., where consumers shop) and equilibrium tax rates depend upon and are *simultaneously* determined by the transactions technologies (the parameters d, D, and E). It is plausible, and can be shown, that no online shopping can occur if E is sufficiently high relative to D, a situation that describes a pre-internet economy.[12] By contrast, if E/D is sufficiently small, it is also plausible, and can be shown, that equilibria exist in which no suburban residents purchase the specialized commodity at the erstwhile central agglomeration point, instead making all of these purchases online. Finally, based on these remarks, it is natural to suppose, and it can be shown, that there are "intermediate" transactions cost configurations that support equilibrium tax policies such that some suburban residents, but not all, purchase the specialized commodity online, while others shop at the city center. This represents a situation, such as is observed in today's economic systems, in which online commerce constitutes a portion – but not all – of those purchases that might possibly be made online. In such "intermediate" cases, changes in fundamental technological parameters (specifically, relative transactions costs) result in

[12] Proving the existence of this and other types of equilibria is non-trivial. Readers concerned with existence questions are referred to the appendix or the working paper version (Agrawal and Wildasin 2019).

changes in the types and locations of commercial activity and, of special interest here, in equilibrium fiscal policies and outcomes.

Using (5), the *necessary* condition for no e-commerce (a "past" regime) is

$$\frac{E}{D} \geq H + \frac{(T_* - t_*)S}{D} \tag{10}$$

while all suburban residents will buy online (a "future" regime) if

$$\frac{E}{D} \leq h + \frac{(T_* - t_*)S}{D}. \tag{11}$$

The intermediate case of some online shopping (a "present" regime) will occur for values of E/D in between. Of course, as explained above, these regimes are determined endogenously by the equilibrium tax rates chosen by the competing jurisdictions, conditional on the technological parameters. Inspecting these equations, intuitively, $E/D \leq H$ refers to the case when the most distance consumer to the city-center will *in the absence of taxes* shop online. Similarly, $E/D \geq h$ refers to the case that the "border" consumer considers buying online *in the absence of taxes*, otherwise we may obtain a condition that causes all hinterland consumers to shop online in the regime where all small jurisdiction residents buy online. Of course, tax differentials will exist in equilibrium and so the necessary conditions delineating each regime can be derived by evaluating these two equations at the Nash equilibrium tax rates. These two conditions are not sufficient because they do not preclude unilateral deviations to discretely higher revenues by one of the jurisdictions.

5 The Effects of Transactions Technologies

In general, all equilibrium properties of the model depend on all of its parameters, and the formal results can accordingly become somewhat notationally complex. In order to focus on the most interesting economic results, the formal expressions stated in the remainder of this section are based on the assumptions that $d = D$, that $S = 1$, and that $H = 1$. The first assumption means that there is only one *relative* transactions cost (the most interesting one), namely, the cost of remote relative to traditional commerce, represented by E/D. The second assumption fixes the relative size of the standardized and specialized commodities in household consumption bundles at unity, i.e., both commodities are equally important. The last assumption normalizes the total population of the metro area. However, these assumptions, which greatly simplify the notation, are inessential for the results. *All* of the results presented in Propositions 1–3 and 5 are valid as stated *without* the imposition of these simplifying assumptions. In particular, the results as stated are valid for all $S \in [0,1]$, and equilibrium tax rates and revenues change smoothly as $S \to 0$, and they encompass, as a special case, the findings of previous studies in which there is no remote commerce.[13]

5.1 Been There, Done That: The World With No Internet

This section begins with the case where the cost of internet access is "prohibitively high", that is, where E/D is so large that no remote purchases occur in equilibrium given by the necessary condition (10) – the situation most similar to the prior literature. The comparison of this case with a scenario where traditional and remote commerce coexist can help to provide guideposts for empirical research, as any dataset of substantial duration will certainly include observations without internet commerce.

Without e-commerce, a case designated by subscript I, the unique Nash equilibrium tax rates are obtained by solving the first-order conditions for the simultaneous solution of (2), using the fact that $\partial B/\partial T = -\partial B/\partial t$ and $\partial b/\partial t = -\partial b/\partial T$, which yields:

$$\begin{aligned} T_I = & \left[\frac{\partial B(T_I,t_I)}{\partial t}\right]^{-1} \times B(T_I,t_I) = D(1+\tfrac{1}{3}h) \\ t_I = & \left[\frac{\partial b(t_I,T_I)}{\partial T}\right]^{-1} \times b(t_I,T_I) = D(1-\tfrac{1}{3}h). \end{aligned} \tag{12}$$

These tax rates are positive, of course, because $1/2 \leq h < 1$ and the large jurisdiction sets a higher tax rate.[14] Thus, closely related to (3), each tax rate is proportional to the equilibrium tax base and inversely proportional to the cross-effect of the other jurisdiction's tax rate on the own-base, or what we may refer to as the "tax base externality." In this case, only B_N and b_N depend on the other jurisdiction's tax rate, so the externality is $1/D$. The corresponding revenues of each jurisdiction in this equilibrium are

$$\begin{aligned} R_I = & \left[\frac{\partial B(T_I,t_I)}{\partial t}\right]^{-1} \times B(T_I,t_I)^2 = \tfrac{1}{D}T_I^2 = D(1+\tfrac{1}{3}h)^2 \\ r_I = & \left[\frac{\partial b(t_I,T_I)}{\partial T}\right]^{-1} \times b(t_I,T_I)^2 = \tfrac{1}{D}t_I^2 = D(1-\tfrac{1}{3}h)^2. \end{aligned} \tag{14}$$

Equilibrium tax revenues thus take a very simple form: the inverse of the tax base "exter-

[13] Interested readers may consult an earlier version of this paper for a detailed presentation of the results and analysis when $d \neq D$, $H \neq 1$ and $S \neq 1$.

[14] These tax rates are distinct from, but deceptively similar to, those in Nielsen (2001): where we use h, Nielsen (2001) has the variable "b", which is equal to $2h-1$ in our notation. Thus, the equilibrium tax rates in his model are

$$\begin{aligned} T = & \; D\left(\tfrac{1}{3}+\tfrac{1}{3}h\right) \\ t = & \; D\left(\tfrac{2}{3}-\tfrac{1}{3}h\right). \end{aligned} \tag{13}$$

nality" times the square of the equilibrium tax base. Making allowance for the more general case in which $S \neq 1$, we have:

Proposition 1. *If the relative cost of remote commerce is sufficiently high, the equilibrium tax rate and total revenues of the large jurisdiction exceed those for the small jurisdiction, while per capita revenues may be either higher or lower. Equilibrium tax rates, tax revenues, and the tax rate differential are increasing functions of the amount S of the specialized commodity.*

The implications of relative jurisdictional size, as represented by h, have been noted by previous authors: large jurisdictions set higher tax rates, obtain more total revenues, and less per capita revenues. These results survive, with the exception of per capita revenues, in the present more general context (i.e., with $S \neq 0$). The equilibrium tax rate differential implies, of course, that some city residents near the jurisdictional boundary choose to shop for the standardized commodity in the smaller jurisdiction. At the same time, the fact that the specialized commodity is available only in the larger jurisdiction means that the city is able to export some of its taxes to non-residents, so that there are two-way shopping flows across the jurisdictional boundary and some of the taxes collected in both jurisdictions are levied on purchases made by non-residents. Furthermore, per capita revenues in the large jurisdiction rise relative to the small jurisdiction as the magnitude of agglomeration rents increases, as represented by S in our model. This explains why per capita revenues may be larger for the big jurisdiction, in contrast to the prior literature where they are smaller in the big jurisdiction (Nielsen 2001).

5.2 Are We There Yet? The Elimination of Traditional Commerce

Instead of supposing that E/D is very large, as in the pre-internet era, one might imagine a world in which remote commerce completely displaces traditional purchases of specialized commodities, corresponding to a very small value of E/D given by the necessary condition (11). In this case, the equilibrium tax rates, identified by the subscript III, are given by

$$\begin{aligned} T_{III} = & \left[\frac{\partial B(T_{III}, t_{III})}{\partial t}\right]^{-1} \times B(T_{III}, t_{III}) = D(\tfrac{2}{3} + \tfrac{2}{3}h) \\ t_{III} = & \left[\frac{\partial b(t_{III}, T_{III})}{\partial T}\right]^{-1} \times b(t_{III}, T_{III}) = D(\tfrac{4}{3} - \tfrac{2}{3}h), \end{aligned} \tag{15}$$

and the corresponding equilibrium revenues of each jurisdiction are

$$\begin{aligned} R_{III} = & \left[\frac{\partial B(T_{III}, t_{III})}{\partial t}\right]^{-1} \times B(T_{III}, t_{III})^2 = \tfrac{1}{D} T_{III}^2 = D(\tfrac{2}{3} + \tfrac{2}{3}h)^2 \\ r_{III} = & \left[\frac{\partial b(t_{III}, T_{III})}{\partial T}\right]^{-1} \times b(t_{III}, T_{III})^2 = \tfrac{1}{D} t_{III}^2 = D(\tfrac{4}{3} - \tfrac{2}{3}h)^2. \end{aligned} \tag{16}$$

All of these numbers are positive and, in particular, $T_{III} > t_{III}$ because $1/2 \leq h < 1$. Making allowance for the more general case in which $S \neq 1$, we have a parallel to Proposition 1:

Proposition 2. *When the relative cost of remote commerce is sufficiently low the equilibrium tax rate and total revenues of the large jurisdiction exceed those for the smaller jurisdiction, but per capita revenue is smaller in the large jurisdiction. Equilibrium tax rates, tax revenues, and the tax rate differential are increasing functions of the amount S of the specialized commodity.*

Despite these parallels with the case where remote commerce is prohibitively costly, it is interesting to note that inter-jurisdictional tax differentials are lower (i.e., $T_I - t_I > T_{III} - t_{III}$) when remote commerce is very attractive. The presence of online sales thus lowers the city's "comparative advantage" in the tax competition game. This result also holds in the more general case where $S \neq 1$. Despite the decline in the large jurisdiction's comparative advantage in the tax competition game, its total revenues remain higher than those of the small jurisdiction. Nevertheless, when online commerce is sufficiently attractive, per capita revenues in the large jurisdiction are lower than those in the small jurisdiction. This result has also been found in prior studies with no agglomeration effects, but note that it contrasts with the ambiguous result shown in Proposition 1 and in Proposition 3 below.

5.3 As Matters Now Stand: The Co-existence of Traditional and Remote Commerce

Let us now turn to the case (designated by subscripts II) where the relative transactions costs of online and traditional commerce, E/D, take on intermediate values at which *both* types of commerce co-exist in equilibrium.[15] Once again we obtain solutions for the Nash

[15] One may wonder what the bounds on E/D are that demarcate this case from the two cases discussed previously. As a partial answer to this question, we have derived the necessary (but not sufficient) condition $E_h < E < E_H$ for existence of an equilibrium when both types of commerce take place in the small jurisdiction. These critical values are obtained by substituting the Nash equilibrium tax rates into (10) and (11) and solving for E. This yields:

$$\begin{aligned} E_h &= \tfrac{D}{2}(4h-1) \\ E_H &= \tfrac{D}{2}(h+2) \end{aligned} \tag{17}$$

Of course, this is a necessary condition. The sufficient condition is implicitly defined by the values of E for which revenues when both types of commerce coexist are greater than the revenues obtained from all possible global deviations, for both jurisdictions. Short of a complete analysis of the existence of equilibrium of all cases, beyond the scope of the present paper, it is not possible to make this sufficient condition explicit. Furthermore, even if explicit, it would provide few economically meaningful insights.

equilibrium tax rates, which again, are always positive:

$$\begin{aligned} T_{II} = & \left[\frac{\partial B(T_{II},t_{II})}{\partial t}\right]^{-1} \times \; B(T_{II},t_{II}) = \frac{D}{2}\left[\frac{2}{3}+\frac{1}{3}\left(h+\frac{E}{D}\right)\right] \\ t_{II} = & \left[\frac{\partial b(t_{II},T_{II})}{\partial T}\right]^{-1} \times \; b(t_{II},T_{II}) = \frac{D}{2}\left[\frac{4}{3}-\frac{1}{3}\left(h+\frac{E}{D}\right)\right]. \end{aligned} \tag{18}$$

Unlike the prior literature, and similar to the case with no internet, this case features two-way cross-border shopping. In particular, cross-border shopping of the specialized good occurs from the small to the large jurisdiction, while cross-border shopping of the standardized good occurs in the reverse direction. However, unlike the case with no internet, a marginal consumer of specialized goods can be influenced by tax rate decisions. As such, two tax bases are in play. In contrast to the previous cases, the tax bases for *both* goods now depend on the other jurisdiction's tax rate. Thus, given that the transport costs of the specialized and standardized bases are equal, the tax base externality is $1/D$ for each base, which summed over both (equally sized) bases is $2/D$. Equilibrium tax revenues are given by

$$\begin{aligned} R_{II} = & \left[\frac{\partial B(T_{II},t_{II})}{\partial t}\right]^{-1} \times B(T_{II},t_{II})^2 = \frac{2}{D}T_{II}^2 = \frac{D}{2}\left[\frac{2}{3}+\frac{1}{3}\left(h+\frac{E}{D}\right)\right]^2 \\ r_{II} = & \left[\frac{\partial b(t_{II},T_{II})}{\partial T}\right]^{-1} \times b(t_{II},T_{II})^2 = \frac{2}{D}t_{II}^2 = \frac{D}{2}\left[\frac{4}{3}-\frac{1}{3}\left(h+\frac{E}{D}\right)\right]^2. \end{aligned} \tag{19}$$

As a parallel to the previous cases, and making allowance for the more general case in which $S \neq 1$, we have:

Proposition 3. *With an intermediate relative cost of remote commerce, some but not all residents of the small jurisdiction purchase the specialized commodity online. The equilibrium tax rate and total revenues of the large jurisdiction exceed those for the smaller jurisdiction, while per capita revenues may be either higher or lower.*

In contrast to the previous cases, this proposition is silent about the effect of changes in the relative importance of the specialized commodity (i.e., S). In fact, although increases in S must raise both equilibrium tax rates when S is sufficiently close to 0, this result may be reversed for S close to 1.[16] The effect of a change in S on revenues is similarly ambiguous.

At this point, one may want to make comparisons across each of the three cases that we consider in order to study discrete (large) changes in E. Interjurisdictional tax differentials,

[16]Relaxing the special assumptions that $S = 1$ and $d = D$, the equilibrium tax rates are

$$\begin{aligned} T_{II} = & \frac{D}{D+dS^2}\left(\frac{1}{3}d[1+h]+\frac{1}{3}dS[\frac{E}{D}-1]+\frac{2}{3}dS\right) \\ t_{II} = & \frac{D}{D+dS^2}\left(\frac{1}{3}d[2-h]+\frac{1}{3}dS[1-\frac{E}{D}]+\frac{1}{3}dS\right). \end{aligned} \tag{20}$$

The relationship between S and these equilibrium tax rates cannot be signed, in general.

the large jurisdiction tax rate, and large jurisdiction revenues will be larger in the "past" (high E/D) than in the "future" (low E/D); the small jurisdiction tax rate and revenues will be smaller in the past than the future. However, it is not possible to make such sharp comparisons of these values when E/D takes on an intermediate value (the "present") as compared with the cases where E/D is very high or very low. When comparing these equilibria, two effects must be taken into account. First, the derivative terms in the expressions above for the equilibrium tax rates ($\frac{\partial B(T,t)}{\partial t}$ and $\frac{\partial b(t,T)}{\partial T}$) are discretely larger in the intermediate case because the jurisdictions are competing over both the specialized and standardized tax bases, intensifying tax competition and thus tending to depress tax rates in that case. Second, however, the equilibrium tax base may increase or decrease as E/D changes discretely, and this may augment or offset the first effect. Thus, the tax rate in the large jurisdiction is unambiguously smaller in the present than the past (in the intermediate case, tax competition is stronger and its tax base is also smaller) whereas when comparing to the future, these two effects work in opposite directions. By similar reasoning, for the small jurisdiction, discrete comparisons of the present with the past are ambiguous, whereas taxes rates are unambiguously smaller in the present than in the future.

In the special case where $d = D$, $S = 1$, and $H = 1$, we can make unambiguous global comparisons of tax rates and revenues for various levels of transaction costs, E/D:

Proposition 4. *Assume that $d = D$, $S = 1$, and $H = 1$. When online shopping costs take on intermediate values, the equilibrium tax rates and revenues are lower than when those costs are either sufficiently high or sufficiently low. When these costs are sufficiently high, the large jurisdiction tax rate and revenues are higher than when these costs are sufficiently low, but the reverse is true for the small jurisdiction. Formally, equilibrium tax rates satisfy $T_I > T_{III} > T_{II}$ and $t_{III} > t_I > t_{II}$. Similarly, equilibrium tax revenues satisfy $R_I > R_{III} > R_{II}$ and $r_{III} > r_I > r_{II}$.*

To derive this proposition, we use the expressions for the Nash equilibrium tax rates and revenues along with (17). Although Proposition 4 focuses on a specific set of parameter values, it provides an important benchmark case that shows how tax rates and revenues need not be *globally* monotonic in E, providing an interesting parallel to Baldwin and Krugman (2004). Technically, this arises because of "corners" (that is, extreme values of E for which all consumers in the small jurisdiction utilize only one mode for purchasing the specialized commodity) in the revenue functions and the discontinuities in their reaction functions. Intuitively, for intermediate values of E, the tax base externality is larger and tax competition is therefore more intense. In the next section, equilibrium tax rates and revenues *do* vary monotonically in E when we restrict attention to *marginal* changes in online shopping costs that do not move the system across "corners". Thus, it would be simplistic to generalize from the analysis of marginal changes to make global comparisons that require attention to the totality of strategic interactions that occur as transaction technologies change.

5.4 Changes in Relative Transaction Costs and Tax Enforcement

The past twenty years have witnessed ever-declining costs and ever-increasing opportunities for online commerce. Policy commentators have suggested that Supreme Court decisions like *South Dakota v. Wayfair* will preserve municipal tax bases in the presence of e-commerce. Recent E.U. reforms taxing digital products at destination have been subject to similar claims. Our model sheds light on the validity of such claims. Specifically, we can investigate changes due to marginal declines in E/D on equilibria where both e-commerce and brick-and-mortar purchases coexist. Keeping in mind that we are interested in a *decline* in E, comparative statics show that:

Proposition 5. *When some but not all residents of the small jurisdiction engage in remote transactions, a decline in the cost of online commerce relative to traditional commerce reduces the equilibrium tax rate and revenues – both total and per capita – in the large jurisdiction but it increases the equilibrium tax rate and revenues in the small jurisdiction. Accordingly, it also reduces the equilibrium tax rate and revenue differentials between the two jurisdictions. In addition, the combined tax revenues of the two jurisdictions decline as the relative cost of online shopping falls.*

The comparative statics of equilibrium tax rates and revenues pick up two effects: a direct one and an indirect or strategic one due to the simultaneous determination of tax rates in this strategic setting. Following Caputo (1996), the total effect of a change in E can be decomposed as

$$\frac{dT_{II}}{dE} = \frac{1}{\Theta}\left(\frac{\partial T}{\partial E} + \frac{\partial T}{\partial t}\frac{\partial t}{\partial E}\right) > 0 \quad \text{and} \quad \frac{dt_{II}}{dE} = \frac{1}{\Theta}\left(\frac{\partial t}{\partial E} + \frac{\partial t}{\partial T}\frac{\partial T}{\partial E}\right) < 0, \tag{21}$$

where $\Theta = 1 - \frac{\partial T}{\partial t}\frac{\partial t}{\partial T} > 0$, as required by stability, and as guaranteed by the specific properties of our reaction functions. The terms in parenthesis consist of the direct effect of a change in E that shifts each jurisdiction's (own) reaction function plus the indirect effect resulting from the shift of the other jurisdiction's reaction function. These two effects work in the *opposite* directions because $\partial t/\partial E = -\partial T/\partial E$ under our assumptions. However, the indirect effect – muted by strategic interaction – is smaller than the direct effect, so the total effect is given by the sign of the direct effect. Thus, for a *decrease* in the cost of e-commerce, we see that the tax rate in the large jurisdiction *falls* and the tax rate in the small jurisdiction *rises*. As shown by (19), a similar decomposition applies for equilibrium tax revenues.

Falling costs of remote transactions put downward pressure on the tax rate and revenues of the large jurisdiction. Perhaps this is unsurprising, as remote commerce erodes the unique locational advantages of the agglomeration and the rents that can be extracted thereby. The effects of increased internet commerce on the small jurisdiction are quite different, however, even though taxes are strategic complements in this case. By expanding new avenues of taxable commerce for the small jurisdiction, increased internet commerce raises its potential tax base. To see how this changes equilibrium policies, suppose hypothetically that both jurisdictions were to hold their tax rates constant as E/D falls. With a fixed tax rate, the tax base and thus the revenues of the small jurisdiction would rise. But a larger base raises the incremental fiscal return to an increased tax rate, and thus the tax rate that maximizes tax revenue for the small jurisdiction must rise. The reverse is true for the large jurisdiction.

Proposition 5 thus sheds new light on the "conventional wisdom" that online shopping, and remote commerce in general, threatens the use of destination-based consumption taxes. This presumption is based partly on the belief that remote transactions offer a path for tax evasion. However, governments have taken important steps over many years to limit tax evasion and, in the U.S., the landscape has recently changed in a major way due to court decisions that reduce tax evasion opportunities; directives in the European setting have had similar effects. Despite these reforms, which effectively enforce taxation of remote sales at destination, the increasing attractiveness of online commerce – because it erodes advantages from initial asymmetries in the spatial distribution of activity – will continue to erode the tax base of some, but not all jurisdictions; it will put downward pressure on some tax rates, but not all; and it will produce revenue "losers" but also "winners."

We may ask, further, about the effect of changes in E on the combined tax revenues of the two governments. Would a (revenue maximizing) "social planner" want to increase or to decrease online shopping costs? To consider this, we add up the equilibrium revenues, assuming that tax rates are set at their Nash equilibrium values. Differentiating with respect to E,[17]

$$\frac{\partial(R_{II}+r_{II})}{\partial E} = - \frac{2}{9}\left(1 - h - \frac{E}{D}\right) > 0 \ . \tag{22}$$

[17]We remind the reader that in this, as in all previous revenue comparisons, we abstract from income effects that might arise due to changes in transaction costs. The assumption of no income effects for taxed commodities means, of course, that any technological changes that increase or decrease purchasing power affect only the demand for the numéraire commodity. As noted in footnote 7, the results continue to hold for sufficiently small income and price elasticities of demand for taxed commodities.

Using the necessary conditions for existence of an equilibrium, this expression, is positive for all possible values of E/D for which such an equilibrium exists. Thus, for a *decrease* in E, aggregate tax revenues *fall*. Even though revenues rise in the smaller jurisdiction, the sum of revenue for both jurisdictions declines because the city tax rate falls, but not by enough to prevent shifting of the specialized good to a lower-tax neighboring jurisdiction. This shifting of the tax base from a higher tax rate to a lower tax rate contributes to a reduction of aggregate revenues. In this sense, falling costs of online commerce turn tax competition into a negative-sum game. This results in a reduction of the ability of central locations to exploit their agglomeration advantages and to engage in tax exporting. For this reason, a revenue-maximizing social planner would wish to increase online transactions costs, although, from a welfare viewpoint, these changes may be advantageous or disadvantageous.

Finally, and critical for empirical analysis, recall that the results highlighted in proposition 5 pertain only to small changes in E, in contrast to the results in proposition 4, which compares equilibrium tax rates and revenues "in the large" as changes in E shift the entire system of commerce from traditional to online shopping, or to a combination of the two. Although proposition 5 indicates that tax rates and revenues vary monotonically with online shopping costs, we have already seen that they need not vary monotonically for "large" changes in E. This is clear from figure 3 which, for specific parameter values, plots the Nash equilibrium tax rates (assuming that they exist) as a function of E. Notice that sufficiently large changes in E may drive the system to discretely different equilibria, resulting in discontinuous and non-monotonic changes in tax rates that are quite different from those that might be anticipated from the comparative statics results for marginal changes in E.

6 Robustness and Extensions

In this section, we present various extensions that highlight the generality of the mechanisms identified in the preceding analysis. As will become clear, the model can be extended from the local application above to many national and international settings. Relevant derivations and formal details are in the online appendix.

6.1 Taxation with Imperfect Enforcement

The canonical model (Kanbur and Keen 1993; Nielsen 2001) assumes effectively that all sales are subject to origin-based taxation. Tax competition for cross-border shoppers would disappear if the destination principle could be perfectly enforced (Agrawal and Mardan 2019, Proposition 1), although, if one maintains the assumption of revenue maximization, this implies that taxes are applied at confiscatory rates. In this respect, the preceding analysis follows the canonical model in assuming that both tax bases are taxed on an origin basis when E/D is high enough to forestall e-commerce. When E/D is sufficiently low that some e-commerce does occur, the origin principle still applies to the standardized good and to any remaining purchases of the specialized good made by cross-border shoppers. At the same time, however, the destination principle is perfectly enforced on any internet (remote) sales, in accordance with current institutional rules in both the United States and the European Union. Thus, tax enforcement at destination in our setting is different from the standard model of Allingham and Sandmo (1972), which features a single audit probability. This is because for some commodities (i.e., those purchased by cross-border shoppers), enforcement of taxes on a destination basis is effectively not possible and the audit probability is zero, whereas destination-basis taxation does occur, with certainty, for other commodities (i.e., those purchased online). While different from standard models of enforcement, this reflects the fact that when consumers remit taxes, tax evasion may be different than when firms remit taxes (Slemrod 2019). Now, we consider imperfect enforcement.

First, we study the case of partial enforcement of the destination principle in the canonical model with no specialized commodities by postulating perfect enforcement of the destination principle on only a fraction of the tax base. This turns out to be closely analogous to the case when e-commerce is virtually costless in our model. We start with a setup similar to Nielsen (2001), but let a denote the fraction of each jurisdiction's population that (knowingly) experiences a comprehensive audit and thus complies with destination taxation. Assume that the fraction a of taxpayers are uniformly distributed across space and that a is identical in both jurisdictions. Anyone who is audited is taxed (but without any additional fines) at the destination rate (detection is perfect). Equilibrium tax rates are then simply equal to those in the canonical model, (13), but now multiplied by $1/(1-a)$.[18] In particular, with $a = 1/2$, the equilibrium tax rates are exactly equal to those obtained earlier for the case where all residents of the small jurisdiction buy the specialized good online, (15). The case where E/D is very low and when specialized purchases per person are equal to standardized purchases ($S = 1$) corresponds to perfect enforcement of destination taxation on half of each consumer's purchases. The equilibrium where E/D is sufficiently low can thus be interpreted as a variation of the canonical model where a fraction a of the base is perfectly audited and a fraction $1-a$ is not audited at all.

Alternatively, consider imperfect enforcement of taxes on online purchases. Some online purchases might be taxed on a destination basis (as previously assumed), others might be

[18]Formally, the equilibrium tax rates are $T = \frac{1}{1-a}D(\frac{1}{3} + \frac{1}{3}h)$ and $t = \frac{1}{1-a}D(\frac{2}{3} - \frac{1}{3}h)$.

taxed at origin by the large jurisdiction, and still others may escape taxation altogether (Goolsbee 2000). To the extent that e-commerce is taxed at origin by the large jurisdiction, the results are similar to those obtained when E is prohibitively high. In the case where e-commerce escapes taxation, reductions in the cost of internet access erode the tax base of the large jurisdiction and only increase the tax base of the small jurisdiction to a limited degree. Such imperfect enforcement puts downward pressure on tax rates and revenues for both jurisdictions. If a large fraction of internet commerce is effectively tax free, the previous conclusion that increased internet commerce raises the tax rate and revenues for the small jurisdiction is reversed. The desire to forestall lost revenue from untaxed e-commerce has undoubtedly encouraged the institutional and regulatory changes, discussed earlier, that have resulted in the establishment of destination taxation as the norm for internet commerce.

6.2 Taxation with Two-way Trade

The baseline model presented above postulates a specialized good found only in one jurisdiction. The model can easily be extended, however, to encompass the case where each jurisdiction is a source for purchases of distinct specialized goods. This variant entails two-way interjurisdictional trade arising from product variety and is thus better adapted to the analysis of tax competition between countries. To represent this case, let S continue to represent the specialized sales from the larger jurisdiction and let s represent specialized sales from the smaller jurisdiction, which (absent remote commerce) can only be purchased from an agglomeration at point $H = 1$ in the small jurisdiction. Focusing on a regime where some residents of the small jurisdiction buy S online and some residents of the large jurisdiction buy s online, the specialized bases become:

$$B_S(T,t) = S\left(\frac{E+(t-T)S}{D}\right) + s\left(1 - \frac{E+(T-t)s}{D}\right) \text{ if } x_S^* \in [h,\, 1],\ x_s^* \in [0,\, h] \quad (23)$$

$$b_S(t,T) = S\left(1 - \frac{E+(t-T)S}{D}\right) + s\left(\frac{E+(T-t)s}{D}\right) \text{ if } x_S^* \in [h,\, 1],\ x_s^* \in [0,\, h] \quad (24)$$

where x_s^* is the cutoff rule, for a person located at point $x \in [0,1]$, such that if $x < x_s^*$, the consumer buys s online. Leaving aside technical questions of existence of such a Nash equilibrium, one can show that the Nash equilibrium will feature $T > t$ if $h \geq 1/2$ and $S \geq s$, with at least one of these inequalities strict. If $h = 1/2$ and $s > S$, then $t > T$ because the spatial size differential vanishes, leaving only the specialized good size differentials. However, as h increases, the relative importance of the standardized good rises and the pattern of equilibrium tax rates cannot be determined because both the relative population size and "net exports" matter and may work in opposite directions.[19] On the other hand, if we revert to the special case with no standardized commodities, one can show that the equilibrium tax rate differential is proportional to $S - s$, as might have been expected. In all cases, jurisdictional asymmetries, whether arising from jurisdictional size differentials, from net exports, or both, play a crucial role in determining the relative levels of each tax rate.

Furthermore, in a model characterized by specialized tax bases in both countries, the signs of the comparative statics with respect to E discussed in Proposition 5 depend on the relative magnitudes of the two specialized bases. In particular, if $S > s$, the results in Proposition 5 continue to hold, but if $S < s$, a decline in the cost of online shopping raises taxes rates and revenues in the "large" jurisdiction and lowers tax rates and revenues in the "small" jurisdiction. Intuitively, the relative magnitudes of the specialized tax bases $(S - s)$ determine which jurisdiction is a "net exporter" of specialized products and the effect of E on tax differentials depends on these trade patterns between the two jurisdictions. Following a decline in E, aggregate revenues of both jurisdictions will decline if $S > s$. Thus, as is now clear, the results in our baseline model, can be interpreted as a model of commodity tax competition at the international level so long as the jurisdiction with the larger population is also a "net exporter" of specialized goods.

The preceding sections have focused on an interpretation in which tax competition occurs between subnational governments within a region given by a central point of agglomeration. Even in the local setting, we have seen that cross-border shopping or trade can occur in both directions in equilibrium. The extension to two points of agglomeration enriches the range of shopping or trade patterns that the model can accommodate within this setting, without fundamentally changing its basic insights. In contrast to the canonical models of commodity tax competition, often thought to be applicable in the international context, our single agglomeration model allows cross-border transactions that are not purely tax driven and the model with two points of agglomerations readily accommodates two way shopping or trade, again, even in the absence of tax differentials. In this respect, it conforms to the international trade flows, including particularly the *intraindustry* trade flows, that have been emphasized in recent theoretical and empirical research on trade.[20] In this respect, country size – as measured by population – is not the only relevant determinant of commercial flows and

[19] Formally, the equilibrium tax rates are $T = \frac{D}{S^2+s^2+1}\left[\frac{2}{3}s + \frac{1}{3}\left(1 + S + h + \frac{E}{D}(S-s)\right)\right]$ and $t = \frac{D}{S^2+s^2+1}\left[\frac{2}{3}(S+1) - \frac{1}{3}\left(h - s + \frac{E}{D}(S-s)\right)\right]$.

[20] Recall the model is agnostic as to whether consumers cross-border shop and travel directly to obtain the specialized good or whether vendors of specialized commodities deliver these goods to consumers in other jurisdictions. The latter of these is what would customarily be referred to as "trade," whether at the international or subnational levels.

equilibrium tax structures. Although tax differentials still matter for the patterns of cross-border transactions of standardized commodities, these tax differentials now additionally depend on other international asymmetries, as represented by the comparative size of the two specialized (or differentiated) product sectors of these jurisdictions, possibly resulting in the smaller population jurisdiction setting a higher tax rate if its specialized sector is especially large. This is a significant generalization of the prior literature.

6.3 Taxation with Differentiated Rates

In the baseline model, we assume that standardized and specialized goods are subject to a common tax rate. In this section, we allow for different commodities to be taxed at different rates. This is easily achieved by choosing each tax rate to maximize the revenue from each tax base separately. This has no impact on the qualitative comparative statics with respect to E although the equilibrium tax rates are affected.[21]

When $S = 1$, the specialized and standardized tax bases are equally important. This implies that the equilibrium tax bases (and tax base externalities) in (18) are a simple (unweighted) sum of those from the partitioned problem. It follows that we can eliminate the standardized base in our model entirely. However, in addition to adding some generality to the model, including the standardized base allows us to link our analysis to the prior literature. In the "past" or "future", it also avoids the unrealistic outcome where governments impose completely confiscatory taxes. This exercise also points the way to generalizations that add many commodities.

6.4 Taxation with Many Commodities

The sharp commodity category groupings above are to some degree arbitrary. At the same time, similar to the "tradable" and "nontradable" distinction in the urban and trade contexts, such stark contrasts can be useful. Although distinctions between goods that can or cannot be purchased online are blurry in practice, data on remote commerce by sector do certainly reveal sharp differences among commodity categories. According to U.S. retail trade data, the fraction of online sales relative to total sales is quite heterogeneous across sectors.

What happens if there is some arbitrary number M of specialized commodities taxed at uniform rates? From section 6.2, in which both jurisdictions have different specialized commodities (a model with three taxed goods plus the untaxed numéraire), it becomes clear that incorporating additional commodities does not introduce dramatically new features to

[21]Formally, the equilibrium tax rates on the specialized base are $T = D(\frac{1}{3} + \frac{1}{3}\frac{E}{D})$ and $t = D(\frac{2}{3} - \frac{1}{3}\frac{E}{D})$. The equilibrium tax rates on the standardized base continue to be given by (13) and are independent of E.

the analysis: the equilibrium tax rates are a function of the jurisdiction's aggregate net exports of specialized goods.

We can generalize still further by considering many heterogeneous products that differ in their transport and online-shopping costs. Suppose that consumers buy S_i units of specialized good $i = 1, ..., M$ and that E_i is the transaction cost incurred when buying this commodity online. As suggested in 6.3, in this situation, the equilibrium tax bases are (weighted) linear combinations of the M specialized commodity tax bases.[22] Thus, in our baseline model, the relative transaction cost term E/D can be interpreted as an "average", weighted by the S_i's, of the commodity-specific transaction costs. As the total of these weights ($\sum_i S_i$) becomes large, the standardized tax base becomes comparatively negligible. In this case, the bulk of transactions taxed on an origin basis would consist primarily of those specialized goods for which the value of E_i is sufficiently high. Then, (small) changes in E_i would only affect tax rates through those goods for which some consumers buy online.

With respect to comparative statics, if $E_i = \epsilon + \zeta_i$ contains a common component ϵ and a product specific component ζ_i, our comparative statics with respect to E can be interpreted as a change in ϵ. The effect of a change in a single ζ_i will have the same sign, but with the magnitude of the effect scaled proportional to the amount purchased of commodity i.

In conclusion, the baseline model can be generalized in many different directions without changing the principal results. These generalizations greatly expand the scope of the model from the local tax competition setting initially studied, to take into account international trade, imperfect tax enforcement, many heterogeneous commodities, product-specific transaction costs, and differentiated tax structures. Thus, the analysis generates insights that extend beyond the initial case. Now, we turn to a further application: profit taxation.

7 An Application to Profit Taxation

The canonical model of cross-border shopping need not only be interpreted solely as applicable for commodity taxes. Remarkably, Keen and Konrad (2013) have shown how the standard linear model can be applied to the analysis of international (or, more generally, interjurisdictional) *profit-shifting behavior* by firms that are subject to profits taxes, a policy

[22] Assume that consumers need to make fixed-cost commodity-specific trips to obtain the specialized goods. Then, assuming that all all specialized goods are purchased by some, but not all suburban residents, the equilibrium tax rates are $T = \frac{D}{1+\sum_i S_i^2}\left[\left(\frac{1}{3}+\frac{1}{3}h\right)+\sum_i S_i\left(\frac{1}{3}+\frac{1}{3}\frac{E_i}{D}\right)\right]$ and $t = \frac{D}{1+\sum_i S_i^2}\left[\left(\frac{2}{3}-\frac{1}{3}h\right)+\sum_i S_i\left(\frac{2}{3}-\frac{1}{3}\frac{E_i}{D}\right)\right]$. This expression can easily generalize further to include two agglomerations, each with many specialized commodities. With different values of E_i, some specialized commodities may never be purchased online, while for others there may never be cross-border shopping. In these cases, the summations above need to be adjusted to account for these "corner" cases.

context quite different from that of cross-border shopping by consumers. Following Keen and Konrad (2013), this section extends the formal apparatus of the preceding sections to the analysis of profit taxation. Just as the model developed above generalizes the standard linear model by adding an urban spatial structure, in the form of an agglomeration point, so now we will show that it can be used to generalize the Keen and Konrad (2013) analysis of profit taxation, shedding additional light on the implications of such taxes in an open-economy context. Specifically, the formal model presented earlier, when applied in the profit-taxation context, can encompass two types of firm responses to taxes: first, the *profit-shifting response* identified by Keen and Konrad (2013), and second, *real relocation responses* by firms who shift not only their accounting profits but their operations from one jurisdiction to another in the presence of tax differentials. This second response arise in the context of our model, where globalization may potentially change the cost of real relocation (as captured by E) differentially from the cost of profit-shifting (captured by d). As will be seen, a decline in d implies that monitoring becomes more difficult, while a decline in E changes the spatial distribution of the tax base in favor of activity that can be more easily monitored.

To present this extension, consider two types of (potentially) multinational firms. The first type of firm has profit-generating operations, organized in two business units (parent/subsidiary) in two countries. The firm's "true" profits in each country are Π in the large country and π in the small country; in order to establish an exact correspondence with the model presented earlier, we set $\Pi = h$ and $\pi = H - h$. Without loss of generality, suppose that $T > t$, which creates an incentive for the unit located in the high-tax country to shift its accounting profits to the low-tax country. Assume that profit shifting is not costless (otherwise, all profits are shifted); as in Keen and Konrad (2013), assume that this cost takes the purely quadratic form

$$c(z) = \frac{dz^2}{2} \tag{25}$$

where z is the amount of profits shifted from the high- to the low-tax country. The firm's net profit is then

$$\pi + \Pi - t(\pi + z) - T(\Pi - z) - \frac{dz^2}{2}. \tag{26}$$

The firm's first-order condition for profit maximization implies that the amount shifted, z^*, satisfies a cutoff rule similar to (4). Under these assumptions, the tax bases from the taxation of this "nonspecialized" (profit-shifting) firm are given by the middle branch of (6) and (7).

The second type of firm *initially* has operations only in the large country. If the cost of multinational operations is sufficiently low, however, the firm may profit from relocating some of its operations to exploit low-tax production opportunities abroad. Let Φ denote the total profits that the firm can earn from its operations, wherever conducted, and let β denote the amount of the firm's profits that derive from its operations in the low-tax country. Letting $C(\beta)$ denote the total cost incurred by the firm when β of its operations take place abroad, its net profits are given by

$$\Phi - T(\Phi - \beta) - t\beta - C(\beta). \tag{27}$$

Let us assume that the cost function $C(\beta)$ takes the linear-quadratic form

$$C(\beta) = e\beta + D\frac{\beta^2}{2} \tag{28}$$

where $e \equiv E - D > 0$ is the linear cost component associated with firm relocation. This functional form generalizes the purely quadratic specification in (25), and it implies that $C(0) = 0$ and $C'(\beta) > 0$, for $\beta \geq 0$, and $C''(\beta) > 0$ for $\beta > 0$. The linear component implies that $C'(0) > 0$, which is to say that the tax differential must be greater than e for there to be any relocation at all. With this function, the firm's choice of β must satisfy

$$\beta^* = \Phi - \frac{E + t - T}{D} \tag{29}$$

so that $x_S^* = \Phi - \beta^*$ is the amount of profit still generated in the home country. If we now set $\Phi = H$ and set $S = 1$, the tax bases in each country attributable to this "specialized" firm are as shown in the middle branches of (8) and (9).[23]

The revenue functions in each country now correspond exactly to those derived in earlier sections, and it follows that the equilibrium tax rates and equilibrium tax revenues also correspond exactly to those derived earlier, with the one added assumption, innocuous in this setting, that $e \equiv E - D > 0$. Needless to say, all of the comparative statics and other results continue to hold. As one interpretation, if we suppose that the "high tax" country corresponds to a developed country, and that the "low tax" country represents a less-developed country, "globalization" may increase profit-shifting opportunities and limit the ability of developed countries to raise revenues from profits taxation, as implied by the Keen and Konrad (2013) analysis. With purely quadratic profit-shifting costs, increased

[23]The "non-specialized" and "specialized" firms in this section obviously correspond to the two sectors of the economy described in preceding sections; these terms are perhaps somewhat apt in this context if we think of firms whose operations initially span two countries as "not specialized" in their locations, whereas firms that are initially operate in only country are "specialized" in the traditional international-trade sense.

"globalization" would correspond to a decrease in the parameter d, i.e., a downward pivot of the marginal cost curve for profit-shifting activities, which puts downward pressure on tax rates in both countries, on the tax differential between them, and on revenues in both countries. Intuitively, the decline in d makes monitoring the tax base more difficult and tax competition is intensified. By contrast, when we extend the model to allow for the relocation of firms from developed to less-developed countries, "globalization" may take the form of a reduction in the parameter E (or e), i.e., the intercept of the marginal cost curve that firms face if they relocate their production activities to other countries. As we know from our previous comparative-statics analysis, a decrease in E – lowers tax rates in developed countries and narrows international equilibrium tax rate differentials, as in the Keen and Konrad (2013) analysis, but it can put *upward* pressure on the equilibrium tax rates and tax revenues of less-developed countries. Intuitively, this arises because a decline in E changes the spatial distribution of the part of the tax base that can be more easily monitored (physical operations), shifting it away from developed countries toward developing countries, reducing international tax differentials. This type of "globalization", then, has quite different consequences from that resulting from increased opportunities for profit-shifting.

Of course, this is a stylized model, so the results are merely suggestive. They do, however, highlight the fact that changes in transactions costs do not always affect fiscal systems in one direction only.

8 Conclusion

We have examined how changes in technology can affect fiscal competition among asymmetrically situated jurisdictions by simultaneously changing the mobility of tax bases while facilitating potentially easier monitoring of those bases by taxing authorities.

We do this first in the context of a model of commodity tax competition with e-commerce. Technological change, represented by an increased propensity to conduct taxable transactions online, affects equilibrium tax rates and revenues. Tax rates and revenues *decrease* in "core" jurisdictions but *rise* in "peripheral" jurisdictions. This results in tax rate convergence, with revenues following a similar pattern, reducing tax exporting and shifting tax revenue towards more outlying areas. Tax revenues fall, in aggregate. The results reflect two potentially conflicting mechanisms. While greater ease of e-commerce increases the mobility of the tax base, it simultaneously facilitates enforcement of destination-based taxes. Due to the initial asymmetry in economic activity, technological change that threatens the tax base in some jurisdictions strengthens the capabilities of others. A decline in the cost of engaging in e-commerce has markedly different effects than a decline in the cost of cross-border shopping, which, as in previous studies, lowers tax rates and revenues for all jurisdictions due to increased tax-base mobility while enforcement capabilities remain unchanged.

We derive these results in a simple spatial model most applicable to tax competition among local governments within a metropolitan area. However, the conclusions of the model continue to hold under many generalizations, making it broadly applicable to commodity tax systems and enforcement at the international or regional level. Furthermore, the model also applies in non-spatial settings, such as the taxation of multinational firms that may shift profits or relocate real business activities. In this case, a decline in the cost of profit shifting makes the tax base more difficult to monitor, placing downward pressure on tax rates in all jurisdictions. By contrast, a decline in the cost of relocating real business activity shifts the interjurisdictional distribution of the tax base that is more readily taxable, resulting in changes in tax rates that are the same as in the commodity tax model.

As the profit-tax application of section 7 suggests, our paper also has some parallels to capital tax competition in the presence of stock effects or agglomeration. In particular, in a dynamic context, existing physical capital ("old" capital) may be very difficult to relocate, but new capital may be highly tax sensitive. In such a setting, a government with a large stock of installed capital (an "agglomeration") can realize a large increase in revenue from increasing its tax rate, but at the cost of discouraging new capital (Wildasin 2003; Wildasin 2000). Thus, analogous to a scenario where E/D is high in our model, jurisdictions with large stocks of "old" capital may maintain high tax rates, possibly facilitating tax differences between capital-rich countries and capital-poor countries (Janeba and Peters 1999; Marceau et al. 2010). Linking our model to this literature, we might think of E as the adjustment cost of capital. As the adjustment cost of capital falls, capital-rich countries would reduce their tax rates because the benefit of taxing "old" capital is dampened by the increased ease with which low-tax countries can attract new investment. In parallel to our results in section 7, we might conjecture that, in the presence of strategic interactions, capital-poor countries could nonetheless raise their tax rates as adjustment costs fall.

Our model also has empirical predictions concerning the effect of online shopping on tax rates. Some empirical research has begun; for example Agrawal (2016) uses internet penetration (the number of internet-service providers) as an exogenous shock to the cost of using the internet. This research shows that having more internet providers places downward pressure on tax rates in large jurisdictions but upward pressure on tax rates in small jurisdictions in states where more firms are remitting taxes, a finding which is consistent with the theoretical predictions discussed above.

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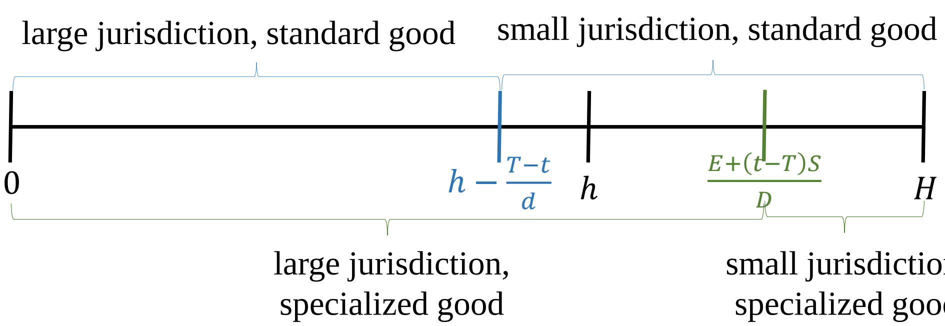
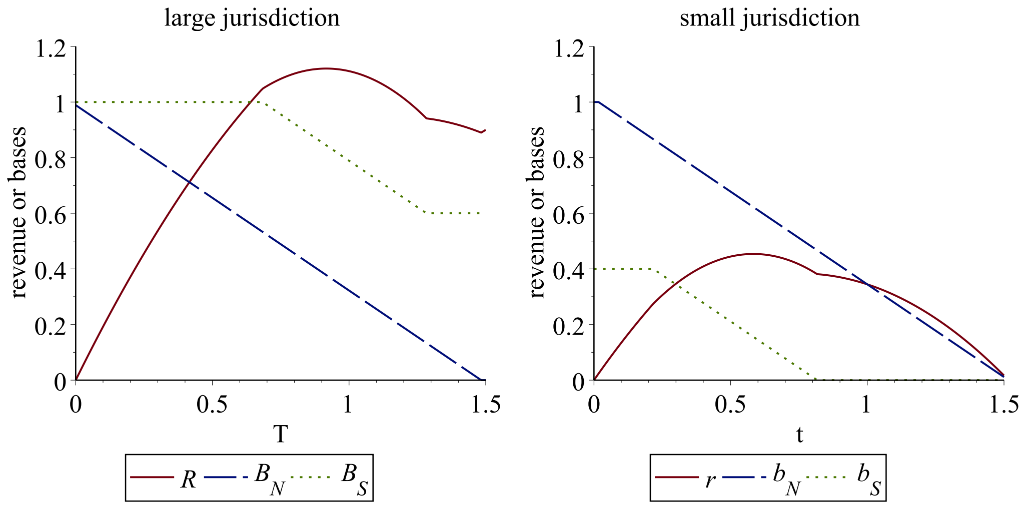
Figure 1: Geography of the Model with Online Shopping



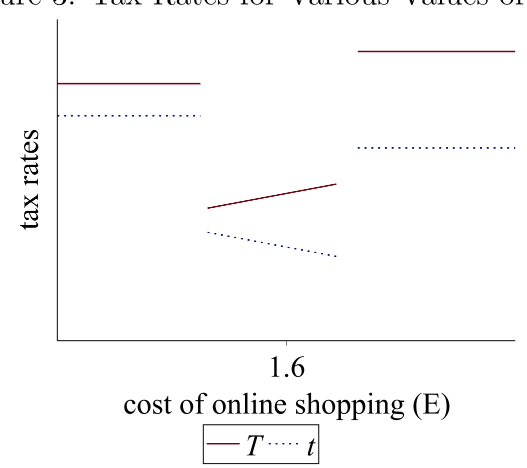
This figure shows the geography of the model. Point 0 is where the specialized firms locate. Then, H is the length of the line segment and h is the distance from the center at which the small jurisdiction starts. The figure shows the cutoff rule for the standardized tax base, which is given by a distance from the center of $h - \frac{T-t}{d}$, assuming that $T > t$. The cutoff rule for the specialized good, $\frac{E+(t-T)S}{D}$, is drawn such that some residents of the small jurisdiction shop online. The model also captures the case of a symmetric two-sided spatial structure, merely by reflection around 0, with a large jurisdiction surrounded by a small jurisdiction on either side.

Figure 2: Example of Revenue Functions when E/D Is of an Intermediate Value



This figure shows the tax revenues for parameter values of $h = 0.6$, $H = 1$, $E = 1.6$, $D = 1.5$, $d = 1.5$, $S = 1$. Each revenue function is drawn assuming the other jurisdiction sets the equilibrium tax rate T_{II} or t_{II}. The size of the specialized tax base and the standardized tax base are depicted with dashed lines. Notice that there is a clear maximum at $T_{II} = 11/12$, $t_{II} = 7/12$, when some (but not all) residents of the small jurisdiction buy online. The figure suppresses portions of the large jurisdiction's revenue function when it has a captive tax base, which can be ruled out using conditions derived in appendix B.

Figure 3: Tax Rates for Various Values of E



This figure shows the equilibrium tax rates and differentials (presupposing existence of equilibrium) for various values of E when other parameter values equal $h = 0.6$, $H = 1$, $D = 1.5$, $d = 1.5$, $S = 1$. There may exist no equilibria in neighborhoods of the points of discontinuity in the tax rates. When $E = 1.6$, an equilibrium does exist in which some (but not all) residents of the small jurisdiction buy online.

9 Appendix: Some Technical Details

As in previous literature, the analysis above depends upon a deliberately stylized model, deriving ultimately from Hotelling's specification of two-agent strategic interactions with a linear spatial structure. For the most part, this specification avoids many technical complexities and offers a tractable framework for the study of tax competition. Nevertheless, the model is sufficiently complex that important technical questions do arise. A brief discussion of these questions, which are discussed in detail in an online appendix and in a previous version of this paper, are warranted here.

Perhaps the most important question concerns existence of equilibrium. Although the best-response functions are well-behaved for the most part, they exhibit discontinuities at points where the tax bases do not respond smoothly to changes in tax rates. At these tax rate pairs (recall, as emphasized previously, that the size of each of the two tax bases for each of the two jurisdictions depends on the size of the tax rate *differentials*), the rate of change of revenues with respect to either tax rate jumps, the revenue functions are not locally concave, and one cannot rely on first-order conditions to characterize the revenue-maximizing policies of each jurisdiction.

It is sometimes possible to appeal to the Topkis (1979) theorem to insure existence of equilibrium in models of this type, but that theorem only holds if payoff functions satisfy the assumption of "antitone differences", equivalent, in the present context, to strategic complementarity. Unfortunately, as explained below, this assumption is not satisfied in the present model, so that no general existence result can be obtained by this avenue. Indeed, one can find specific values for the fundamental parameters of the model for which no equilibrium is possible. Nevertheless, we have verified that there are substantial ranges of parameter values – notably allowing for values of S throughout the range $0 \leq S \leq 1$ – for which existence is indeed guaranteed.

To see why existence problems arise, recall that the tax bases described in (6)-(9) are continuous and piecewise linear in both tax rates. As a consequence, over a *portion* (the "interior") of its domain, each jurisdiction's revenue function is a negative quadratic function of its own tax rate and a positive linear function of the other jurisdiction's tax rate. Ignoring corner conditions at which either of the two bases is at a maximum or minimum value, it is a simple matter to find the tax rates that maximize these revenue functions and to show that the best-response function of each jurisdiction is an increasing linear function of the other jurisdiction's tax rate in the neighborhood of such a revenue maximum.

Existence of equilibrium, however, requires that each jurisdiction maximizes its revenues over *all* non-negative tax rates, and this necessitates consideration of "corner" situations at which either of the two bases, for either of the two jurisdictions, is at a maximum or minimum, corresponding to the upper and lower branches of the expressions in (6)-(9). The best-response functions are discontinuous at these corners. One might hope to appeal to the Topkis theorem (Topkis 1979) to prove existence. However, Topkis' theorem depends on the assumption of "antitone differences," which stated in the notation of our model, means that

$$R(T', t') - R(T, t') \geq R(T', t) - R(T, t) \tag{30}$$

for all $T' > T$ and $t' > t$. Unfortunately, one can show that this condition is *not* satisfied in our model. Even though this condition does hold at all points of continuity for the best-response functions, it does not hold globally.

In addition, considering that demand functions are treated as price-inelastic, equilibria can fail to exist because some households (those in the city) cannot escape local taxes even at very high tax rates, so that revenue functions are potentially unbounded. Thus, existence also depends on a technical assumption discussed in the online appendix, namely that demand for the specialized commodity becomes sufficiently elastic at a threshold price[24] corresponding to a tax configuration where the large jurisdiction's tax rate is so high that all of its residents purchase the standardized commodity in the small jurisdiction (i.e., $B_N(T, t) = 0$) and that all residents of the small jurisdiction purchase the specialized commodity online (i.e., $B_S(T, t) = Sh$). This assumption rules out an otherwise explosive (and economically uninteresting) part of the revenue function for the large jurisdiction where it can raise its revenues without limit by imposing arbitrarily high tax rates on its "captive" resident purchasers of the specialized commodity.

These technical problems can be surmounted, in part, as discussed in the online appendix.

[24]More formally stated, let P_S denote the tax-inclusive price of the specialized good and let $\bar{P}_S$ denote a threshold price. Then, each household's demand for the specialized goods is given by $S(P_S)$, where $S(P_S) = S$ for all $P_S \leq \bar{P}_S$ and where, for all $P_S \geq \bar{P}_S$, we assume that $|\epsilon_S| := |d\log S(P_S)/d\log P_S| \geq 0$ is "sufficiently large". As discussed in the online appendix, $\epsilon_S \geq 1$ is sufficient (but not necessary) for our results.

Online Appendix for Technology and Tax Systems

David R. Agrawal and David E. Wildasin

10 Online Appendix A

A.1 Institutional Details

Taxation of cross-border sales. In the United States, cross-border shopping is legally taxed at destination, but due to enforcement challenges, these sales are effectively origin rated. Such a contrast arises because of the structure of the sales and use tax system. When a cross-border shopper purchases a good in a lower sales tax jurisdiction, the shopper is legally required to report this purchase to the tax authority. Upon declaration of the purchase, the individual is required to pay the use tax (receiving a credit for sales taxes already paid, to avoid double taxation). Thus, destination-based use taxes should be remitted by *purchasers* to the tax authorities in their home jurisdictions, provided that the purchases are made in a lower sales tax jurisdiction.[25] In the presence of open state and local borders, cross-border purchases are hard to detect by the tax authority. For this reason, the use tax is notoriously evaded, and with the exception of registered goods, such as cars, cross-border sales are effectively origin rated.

Similar origin based taxation arises for E.U. cross-border sales. For example, within the European Union, the origin-principle generally applies on physical cross-border transactions. Even for cross-border purchases between member and non-member states, the origin principle may apply due to *de minimis* rules (Art. 31 Directive 2006/112/EC). Consider as an example from Agrawal and Mardan (2019) of cross-border shopping from a resident outside of the EU to an EU country: "a Swiss resident who engages in cross-border shopping in Germany. Generally, the Swiss resident can ask for a VAT refund slip to get a full rebate on his VAT paid after approval at German customs. If the value of the purchase is less than the de minimis exemption, then no tax liability is due upon the importation of the purchase. If the Swiss resident exceeds the de minimis threshold, he still gets the full rebate approved at German customs but is legally required to declare his purchase at Swiss customs and pay the Swiss taxes. However, by not declaring his purchase he can evade the additional tax payments if Swiss customs fails to detect the individual."

Taxation of e-commerce. In the U.S., the direct imposition of state and local sales and use taxes on remote vendors has been limited by judicial rulings. In particular, the U.S. Supreme Court, in *Quill Corp. v. North Dakota* (504 U.S. 298 (1992)), found that states and localities exceed their constitutional taxing powers if they require "out-of-state" vendors to remit taxes on sales to their residents, unless the vendors have sufficient "nexus" (including a physical presence requirement) with the taxing state. Because nexus is a state-level concept, taxes (both state and local) on online transactions that feature a buyer and an online seller

[25] See Agrawal and Mardan (2019). Although sales and use taxes may be different, they are equal in most states. States also differ in how they legally treat within-state cross-border shopping.

in the same state are remitted by the seller and the destination (local) tax rate prevails. The upshot is that only taxes on transactions between a buyer and a seller located in two *different* states need to be remitted by the buyers. Historically, however, this was the bulk of online transactions.[26] Recently, the United States Supreme Court entirely changed the taxation of remote sales in its recent decision, *South Dakota v. Wayfair, Inc.* (585 U.S. ___ (2018)). This ruling abolishes the physical presence standard and instead allows states to compel vendors to remit taxes at destination if they have significant economic and virtual contacts in the state. Such a rule means that, absent large online vendors changing their retail strategies, most online transactions will be sourced at destination as states update their laws.

These online shopping rules mirror those established by the European Union. The place of supply of goods and services under E.U. VAT is not easy to describe given it depends on many characteristics of the transactions and seller (Hellerstein 2005; Hellerstein and Gillis 2010). In response to the "enormous changes in the volume and pattern in trade and services" and after years of political negotiations, the EU reached an agreement to tax at the place of consumption under a distance selling rule where suppliers with a sufficiently large number of transactions to a member state are required to remit taxes to the country where consumption tax place (Art. 33 Directive 2006/112/EC) At the same time, the E.U. maintains the origin principle for physical cross-border purchases internal to the E.U. common market.[27] More recently, the E.U. has passed laws that move toward destination taxation on digital products such as e-books or video streaming (Agrawal and Fox 2017).[28] Thus, similar to the recent Supreme Court ruling, online transactions from vendors with significant economic presence – as measured by sales – in the member state are taxed at destination, while cross-border transactions remain taxed at origin.

A.2 Additional Works Cited

Hellerstein, Walter. 2015. "A Hitchhiker's Guide to the OECD's International VAT/GST Guidelines." *Florida Tax Review*, 18(10): 589–637.

Hellerstein, Walter, and Timothy H. Gillis. 2010. "The VAT in the European Union." *Tax Notes*, 127: 461–471.

[26]However, given the structure of our model, even in the pre-*Wayfair* era, at the local level our model still has substantial applicability. The key here is that, under *Quill*, whether a firm collects taxes at destination is a result of whether the firm has nexus in a state. In this way, our model would apply under *Quill* if we consider both towns in the same state. This is because the online sales originate from the internet vendor that sells specialized commodities, but because this vendor has a physical presence (a store) in the large locality, it must collect and remit appropriate taxes at destination for online sales (though it collects and remits origin-based taxes for cross-border shopping).

[27]As in the U.S., if sales to member state are below a threshold, the origin principle may apply (Art. 34 Directive 2006/112/EC).

[28]See (Art. 5 Directive 2008/8/EC, Art. 58 and Annex II Directive 2006/112/EC).

B Online Appendix B

The text presents comparative-statics results showing how key endogenous variables – tax rates, revenues, and others – vary with respect to critical parameters in different equilibria. Such results, of course, require existence and possibly uniqueness of equilibria. The problems of existence and uniqueness of equilibria raise many technical complexities that are often sidestepped in the literature (for example, assuming existence of an equilibrium), or that are finessed by symmetry and other strong assumptions. Because this paper develops a completely novel model, it is perhaps especially important to address these technical questions. This appendix analyzes the existence and uniqueness of equilibrium for a wide range of the model's parameter values, encompassing many economically interesting cases. We hasten to note, however, that we do not provide a general existence proof for all possible parameter values (see the appendix in the text for why we cannot apply Topkis' Theorem). Indeed, it can be verified – by example – that no such proof is possible. Here, we focus on model with $H = 1$, $d = D$, and $S = 1$.[29] Our analysis of existence and uniqueness is not exhaustive, but the results presented here provide a firm foundation for the comparative statics analysis results presented in the paper, and demonstrate that the basic modeling approach raises no insuperable technical obstacles.

B.1 Preliminaries

As might be expected, and as is shown below, the existence of one type of regime or another is highly parameter-dependent. For example, for fixed values of other parameters, the cost of online transactions can be made prohibitively high (the "past") by making the cost parameter E sufficiently large, whereas online transactions can be made costless by setting $E = 0$ (the "future"). Thus, to obtain conditions for the existence of type II equilibria, it is plausible – and we show below – that E must take on values in some intermediate range, the limits of which depend on the other parameters of the model.

As a matter of notation, let $\pi = (h, H, d, D, E, S) = (h, 1, D, D, E, 1)$ denote the full set of model parameters. Two other elements, mainly of technical importance, must also be specified.

First, households must have sufficient resources to survive or, equivalently stated, they must have non-zero surplus. This condition, which need not discussed further, can always be satisfied, for any possible configuration of parameters.[30]

Second, in order to avoid economically-uninteresting pathological cases, the demand for the specialized goods must become price elastic above some threshold price. In the absence of this condition, the large jurisdiction could raise its tax rate indefinitely, collecting more and more taxes from its own residents whose specialized goods purchases are always subject to its tax. In order to avoid this unrealistic and uninteresting outcome, while maintaining

[29] See Agrawal and Wildasin (2019) for an existence proof that relaxes the first two assumptions and proves existence for $S \in [0, 1]$, given other parameters.

[30] This condition is certainly met if household incomes are sufficient to cover all consumption expenditures, transactions costs, and taxes under any possible spatial and policy configuration.

the simplicity of the assumption of inelastic demand where possible, we assume that there is some threshold price $\bar{P}_S$ above which the demand for specialized goods becomes "sufficiently elastic", and below which (i.e., "in the relevant range") it is perfectly inelastic, where the tax-inclusive price is denoted by P_S, given by $1+T$ for city residents and by $1+t$ for residents in the small jurisdiction. More formally stated, each household's demand for the specialized goods is given by $S(P_S)$, where $S(P_S) = S = 1$ for all $P_S \leq \bar{P}_S$ and where, for all $P_S \geq \bar{P}_S$, we assume that $\epsilon_S := d\text{log}S(P_S)/d\text{log}P_S \leq 0$ is "sufficiently small" (i.e., sufficiently elastic). As discussed further below, $\epsilon_S \leq -1$ is sufficient (but not necessary) for our results.[31] This elasticity condition is "technical" in the sense that it pertains to portions of the demand function that are never observed in equilibrium, but some version of it is needed to rule out the possibility that a jurisdiction could hypothetically raise its revenues indefinitely through an ever-increasing tax rate applied to a perfectly price-inelastic and captive tax base.

B.2 Nash Equilibrium: A Formal Definition

Under the assumption that some but not all small jurisdiction households purchase the specialized good online, and some households purchase the standardized good in both jurisdictions, each jurisdiction's revenue function is inverse quadratic in its own tax rate and its best-response function, in the neighborhood of the candidate Nash equilibrium, is a linear increasing function of the other jurisdiction's tax rate with a slope of 1/2; the unique intersection of the best-response functions determines our *candidate* Regime-II Nash equilibrium tax rates, $(T_{II}[\pi], t_{II}[\pi])$, given by (18). These tax rates are derived assuming that both jurisdictions tax both bases and some residents of the small jurisdiction shop online so that, in the neighborhood of the equilibrium, are given by the middle branches of (6)-(9).

However, the revenue function given only by these middle branches does not capture all of the possible shopping patterns of the specialized or standardized bases. To encompass all possibilities simultaneously, it is helpful to express each of the two components of the tax base for each jurisdiction, that is, the volume of taxable sales of the standardized good and the volume of taxable sales of the specialized good, in a general form.

For sufficiently low or sufficiently high tax rates, a jurisdiction can attract or repel all transactions involving the standardized good; similarly, there are lower and upper rates at which a jurisdiction attracts or repels maximal amounts of transactions involving specialized commodities. These tax rates, denoted by $\underline{\Gamma}_N$, $\overline{\Gamma}_N$, $\underline{\Gamma}_S$, $\overline{\Gamma}_S$ for the large jurisdiction and by $\underline{\Gamma}_n$, $\overline{\Gamma}_n$, $\underline{\Gamma}_s$, $\overline{\Gamma}_s$ for the small jurisdiction, maximize or minimize the respective tax bases for each jurisdiction. These equations, already presented in the text, are reproduced here using

[31] "Specialized goods" may be interpreted as an aggregate of many goods, each possibly with its own demand function, dependent on its own price. By normalization of units, we may set each of these per-unit prices equal to 1. The household decision to purchase any one of these goods, and the amount to be purchased, depends on its tax-inclusive price P_S. It is independent of transactions costs, however, which are an indivisible overhead cost of market access, except when (and if) demand vanishes completely.

this notation. For the standardized good,

$$B_N(T,\, t,\, \pi) = \begin{cases} 1 & \text{if} \quad T \leq \underline{\Gamma}_N := t - D(1-h) \\ h + \frac{t-T}{D} & \text{if} \quad T \in [\underline{\Gamma}_N, \overline{\Gamma}_N] \\ 0 & \text{if} \quad T \geq \overline{\Gamma}_N := t + Dh. \end{cases} \tag{B.1}$$

Because $B_N + b_N = H$, we may equivalently define

$$b_N(t,\, T,\, \pi) = \begin{cases} 0 & \text{if} \quad t \geq \overline{\Gamma}_n := T + D(H-h) \\ 1 - h + \frac{T-t}{D} & \text{if} \quad t \in [\underline{\Gamma}_n, \overline{\Gamma}_n] \\ 1 & \text{if} \quad t \leq \underline{\Gamma}_n := T - Dh. \end{cases} \tag{B.2}$$

For the specialized good, the tax base of the large jurisdiction is

$$B_S(T,\, t,\, \pi) = \begin{cases} 1 & \text{if} \quad T \leq \underline{\Gamma}_S := t + E - D \\ \frac{E}{D} + \frac{t-T}{D} & \text{if} \quad T \in [\underline{\Gamma}_S, \overline{\Gamma}_S]; \\ h & \text{if} \quad T \geq \overline{\Gamma}_S := t + E - Dh; \end{cases} \tag{B.3}$$

for the small jurisdiction, the base is

$$b_S(t,\, T,\, \pi) = \begin{cases} 0 & \text{if} \quad t \geq \overline{\Gamma}_s := T - (E-D) \\ 1 - \left(\frac{E}{D} + \frac{t-T}{D}\right) & \text{if} \quad t \in [\underline{\Gamma}_s, \overline{\Gamma}_s]. \\ 1 - h & \text{if} \quad t \leq \underline{\Gamma}_s := T - (E - Dh)\,. \end{cases} \tag{B.4}$$

In both cases, demands for the specialized goods are given by the demand function $S(P_S)$ which, strictly speaking, depends on the prices faced by households in each jurisdiction and thus on the tax rates. However, in order to simplify writing, and in accordance with the elasticity assumption mentioned above and discussed formally below, we may suppose that the threshold price $\bar{P}_S$ is sufficiently large that $S = 1$ may be treated as a constant in these expressions and in related expressions below.

Each of these expressions has been written so as to emphasize that the tax bases of each jurisdiction depend, first, on its own tax rate, and secondly, on the other jurisdiction's tax rate and on other parameters of the model. Observe that the functions $B_N(T; t, \pi)$ and $B_S(T; t, \pi)$ are each continuous and piecewise linear functions of the tax rates, decreasing in T and increasing in t, and likewise (*mutatis mutandis*) for $b_N(t; T, \pi)$ and $b_S(t; T, \pi)$.

We may now define *general* revenue functions, for all tax rates $(T, t) \in \mathbb{R}^2_+$ and for all parameter values $\pi \in \mathbb{R}^6_+$ as

$$R^G(T; t, \pi) = T\left(B_N[T; t, \pi] + B_S[T; t, \pi]\right) \tag{B.5}$$
$$r^G(t; T, \pi) = t\left(b_n[t; T, \pi] + b_s[t; T, \pi]\right). \tag{B.6}$$

These revenue functions account for shopping patterns other than those assumed for the case when E/D takes on an intermediate value. To tackle the question of existence, observe, first, that each general revenue function is *continuous* in both tax rates and in all of the parameters. These revenue functions are also piecewise diffentiable, but *not* continuously differentiable, in the tax rates and parameters. The reason for this is obvious from (B.1), (B.2), (B.3), and (B.4): each component of the tax bases varies continuously but not differentiably with the tax rates and parameters. For this reason, and in contrast to simpler models with only one tax base, no agglomerated spatial structure, and only one type of transactions technology, there is no *a priori* guarantee that each jurisdiction has a continuous best-response function. There is therefore no guarantee that a Nash equilibrium exists at all, and, if there is a Nash equilibrium, that it satisfies the conditions for regime II. It is therefore a non-trivial task to show that there are some regions of the parameter space for which a regime-II equilibrium does exist.

B.3 Finding a Regime II Equilibrium

To begin with, it is obvious that there are values of the key technology parameter E for which such an equilibrium is not possible; it is plausible to conjecture, and we now show, that a regime-II equilibrium – the regime between the "past" and the "future" – can only occur for "intermediate" values of the parameter.

B.3.1 The Range $[E_h, E_H]$ of Admissible Values of E

Whether a regime II equilibrium can occur clearly depends on the cost of accessing the internet, E. In order to insure that some but not all small jurisdiction residents choose to purchase online, it must be the case that $h \leq x_S^* \leq H$ when $(T, t) = (T_{II}, t_{II})$. Because

$$x_S^* = \frac{E}{D} - \frac{T_{II} - t_{II}}{D}, \tag{B.7}$$

we have

$$x_S^* = \frac{1}{3}\left(1 - h + \frac{2E}{D}\right). \tag{B.8}$$

Observe that x_S^* is linearly increasing in E and we may therefore solve (B.8) for the values of E at which x_S^* reaches its lower and upper bounds of h and $H = 1$ for regime II:

$$E_h = \frac{1}{2}\left(4h - 1\right) D \tag{B.9}$$

and

$$E_H = \frac{1}{2}\left(h + 2\right) D. \tag{B.10}$$

These expressions, define lower and upper bounds on the parameter E, showing that for any configuration of other parameters, no type-II equilibrium can exist unless $E_h \leq E \leq E_H$. This is of course a *necessary* condition for equilibrium, not a *sufficient* one.

We note from (B.9) and (B.10) that

$$E_H - E_h = \frac{3}{2}(1-h)D > 0 \tag{B.11}$$

i.e., $[E_h, E_H]$ is a non-degenerate interval. In establishing restrictions on parameters sufficient for existence of an equilibrium for regime II, we may henceforth limit attention to values of $E \in [E_h, E_H]$.

B.4 Existence of Equilibrium with Both Transaction Technologies

To show existence, we first demonstrate that a Regime II equilibrium exists for one *particular* set of parameters, namely, $\pi^0 := (h^0, H^0, d^0, D^0, E^0, S^0) = (3/5, 1, 3/2, 3/2, 8/5, 1)$. In this analysis, we focus on the case discussed in the text where $S = 1$, $H = 1$ and $d = D$. The interested reader may consult prior versions of our working paper where, we allow for S to take any value between 0 and 1 and we allow $d \neq D$. This specific set of parameter values provides a starting point from which it will follow easily that equilibria also exist in a neighborhood of π^0 in $\mathbb{R}^6_+$ with positive measure.

To begin, consider the tax rates $(T_{II}(\pi^0), t_{II}(\pi^0))) := (T^0, t^0)$ obtained from the Regime-II best-response functions, shown in (18), for $\pi = \pi^0$. These are the unique solutions to the "restricted" revenue-maximization problem when all tax bases are at an interior solution. It can easily be shown that $T^0 \in \left[\underline{\Gamma}_N, \overline{\Gamma}_N\right]] \cap [\left[\underline{\Gamma}_S, \overline{\Gamma}_S\right]$ and $t^0 \in \left[\underline{\Gamma}_n, \overline{\Gamma}_n\right]] \cap [\left[\underline{\Gamma}_s, \overline{\Gamma}_s\right]$. Then, to show that these are Nash equilibrium tax rates we need to show that T^0 maximizes $R^G(T, t^0)$ for all $T \geq 0$ and that t^0 maximizes $r(t, T^0)$ for all $t \geq 0$.

B.4.1 Large Locality

To check that these conditions are satisfied for our parameter values, consider first the large jurisdiction. Observe that R^G the revenue function for regime II coincide for all $T \in [\underline{\Gamma}_N, \overline{\Gamma}_N] \cap [\underline{\Gamma}_S, \overline{\Gamma}_S]$. It remains to show that $R^G(T^0, t^0) \geq R^G(T, t^0)$ for all $T \in [0, \max\{\underline{\Gamma}_N, \underline{\Gamma}_S\}] \cup [\min\left\{\overline{\Gamma}_N, \overline{\Gamma}_S\right\}, \infty]$. The first of these intervals represents possible "downward deviations" by the large jurisdiction, in which it selects a tax rate sufficiently small that it captures the maximum feasible amount(s) of one or both of the tax bases; the second represents possible "upward deviations" in which it chooses a tax rate sufficiently high that it retains only the minimum feasible amount(s) of one or both of the two bases.

- *Downward Deviations:* $T \in [0, \max\{\underline{\Gamma}_N, \underline{\Gamma}_S\}]$

For $\pi = \pi^0$, $\max\{\underline{\Gamma}_N, \underline{\Gamma}_S\} = \max\{t^0 - 3/5, t^0 + 1/10\} = t^0 + 3/5$. For any $T \leq t^0 + 1/10$, $B_N \leq h^0$ and $B_S = 1$, and therefore $R^G(T, t^0) \leq T(1 + h^0) \leq (t^0 + 1/10)(1 + h^0)$ for all downward deviations. Explicit calculations show that

$$R^G(T^0, t^0) - (t^0 + 1/10)(1 + h^0) = \frac{73}{270} > 0.$$

Thus, the large jurisdiction cannot increase its revenues by a downward deviation from T^0.

- *Upward Deviations:* $T \in [min\left\{\overline{\Gamma}_N, \overline{\Gamma}_S\right\}, \infty]$.

We must show that the large jurisdiction cannot raise its revenues by an upward deviation.

Beginning at $T = T^0$, increases in T cause both $B_N(T, t^0)$ and $B_S(T, t^0)$ to decline. At sufficiently high values of T, one or the other of these bases reaches its minimum value, i.e., either $B_N = 0$ or $B_S = 3/5$ (or possibly both), as the case may be.

Because $\overline{\Gamma}_N - \overline{\Gamma}_S = 1/5$, $\min\left\{\overline{\Gamma}_N, \overline{\Gamma}_S\right\} = \overline{\Gamma}_S$. In this case, $B_S(T, t^0) = h^0 = 3/5$ for all $T \geq \overline{\Gamma}_S$ and thus $R^G(T, t^0) = T(3/5) + TB_N(T, t^0)$, which is the sum of a positive linear function of T and a negative quadratic function of T, up to the value $T = \overline{\Gamma}_N$. For $T \geq \overline{\Gamma}_N$, $R^G(T, t^0) = T(3S/5)$.

For $T \leq \overline{\Gamma}_N$, the maximum of $R^G(T, t^0)$ occurs at $T = \hat{\hat{T}}$ at which

$$\frac{\partial R^G(T, t^0)}{\partial T} = 0,$$

provided that $T = \hat{\hat{T}} \leq \overline{\Gamma}_N$. Solving this condition explicitly, we find that $\hat{\hat{T}} < \overline{\Gamma}_S < \overline{\Gamma}_N$, and therefore $R^G(T, t^0)$ is a decreasing function of T for all $T \in [\Gamma_S, \Gamma_N]$. It attains its maximum, over this interval, at $T = \overline{\Gamma}_S$. We therefore calculate

$$R^G(T^0, t^0) - R^G(\overline{\Gamma}_S, t^0) = \frac{121}{675} > 0.$$

Once again, therefore, we see that the large jurisdiction cannot increase its revenue by raising its tax rate to any value above T^0 before its tax base falls to its minimum value of $B_S(T, t^0) = 3/5$. At this point, revenue again becomes a positive linear function of the tax rate, namely $R^G(T, t^0) = T(3/5)$, and, of course, this exceeds the amount of revenue in the regime-II Nash equilibrium for sufficiently high values of T. We next discuss this possibility.

- *The Determination of* $\overline{P}_S$

The argument so far has shown that the large jurisdiction cannot increase its revenue by an upward deviation from its regime-II tax rate, except by raising the tax rate to such a high level that $B_N(T, t^0) = 0$ and $B_S(T, t^0) = 3/5$, at which point $R^G(T, t^0) = T(3/5)$.

It is at this point that we invoke the condition that the demand for the specialized goods becomes sufficiently elastic at the threshold price of $\overline{P}_S$. It remains to be shown that there exists such a threshold. Given that $\pi = \pi^0$, this can easily be determined, for $S = 1$, by setting $\overline{P}_S(\pi^0) = 1 + \max[\overline{\Gamma}_N(\pi^0), \overline{\Gamma}_S(\pi^0)]$. Above this threshold, we impose the condition discussed in section B.1, namely that the elasticity of demand for the specialized good is "sufficiently elastic." We can now make clear that "sufficiently elastic" means that revenue is a declining function of the tax rate for $1+T > \overline{P}_S(\pi^0)$. This condition is clearly satisfied if the absolute value of the elasticity of demand is greater than unity. It is clear that this threshold price is sufficiently high such that every household is on the perfectly inelastic portion of its demand curve for the specialized commodity for all regime II equilibrium tax rates, which now justifies why we have ignored demand variations in the preceding analysis.

Although a condition like $\epsilon_S < -1$ for $P_S > \overline{P}_S$ is sufficient for existence, it is certainly not necessary. Letting $\overline{T}$ be the tax rate that achieves $\overline{P}_S$, because $R^G(\overline{T}, t^0) < R^G(T^0, t^0)$, it is possible for R^G to increase, at least somewhat, for tax rates in excess of $\overline{T}$. Necessity merely requires that $R^G(T, t^0) \leq R^G(T^0, t^0)$ for all $T \geq \overline{T}$. The condition that $\epsilon_S < -1$ suffices to insure that this is true because it means that revenue declines monotonically for $T > \overline{T}$, but it is not necessary to insist on monotonicity: the revenue function could be mildly increasing – i.e., the demand functions for the specialized goods could exhibit ranges with $\epsilon_S > -1$ – through one or some ranges of T values above $\overline{T}$. These are quite weak restrictions on the demand functions.

B.4.2 Small Locality

The situation for the small jurisdiction is easier to analyze because it has no "captive" tax base. Once again, it is necessary to consider both downward and upward deviations from the regime-II Nash tax rate of $t = t^0$, keeping T fixed at T^0.

- *Downward Deviations:* $t \in [0, \max\{\underline{\Gamma}_n, \underline{\Gamma}_s\}]$

Beginning at $t = t^0$, reductions in t cause both $b_N(t, T^0)$ and $b_S(t, T^0)$ to rise. At sufficiently low values of t, one or the other of these bases reaches its maximum value, i.e., either $b_N = 1$ or $b_S = 2/5$ (or possibly both), as the case may be.

Because $\underline{\Gamma}_n - \underline{\Gamma}_s = -(1/5)$, $\max\{\underline{\Gamma}_n, \underline{\Gamma}_s\} = \underline{\Gamma}_s$. The tax base can be no greater than $1+2/5$ and the tax rate can be no greater than $\max\{\underline{\Gamma}_n, \underline{\Gamma}_s\}$. The revenue from a downward deviation is thus no greater than the upper limit $r_B = \underline{\Gamma}_S(1 + 2/5)$. This limit can be compared to equilibrium revenue in regime II, i.e., to $r(t^0, T^0)$. Explicit calculations show that

$$r(t^0, T^0) - r_B = \frac{203}{1350} > 0$$

Thus, the small jurisdiction cannot increase its revenues by a downward deviation from $t = t^0$.

- *Upward Deviations:* $t \in [\min\{\overline{\Gamma}_n, \overline{\Gamma}_s\}, \infty]$.

Note first that $\min\{\overline{\Gamma}_n, \overline{\Gamma}_s\} = \overline{\Gamma}_s$. Hence, for any upward deviation, $b_s(t, T^0) = 0$, and therefore $r^G(t, T^0) = tb_n(t, T^0)$. Let $\hat{t}_n = \text{argmax}_{<t>} tb_n(t, T^0)$. Solving explicitly for this tax rate,

$$\hat{t}_S = \frac{91}{120},$$

Even though $b_n(\hat{t}_S, T^0)$ may be positive, we can use $\hat{t}_S$ because any other tax rate solving this problem with a constraint will produce even lower revenue. The maximum possible revenue that can be obtained by taxing only the specialized goods is

$$\hat{t}_S b(\hat{t}_S, T^0) = \frac{8281}{21600}.$$

Again calculating explicitly, for any $t \geq \overline{\Gamma}_n$,

$$r^G(t^0, T^0) - r^G(t, T^0) \geq r^G(t^0, T^0) - \hat{t}_n b_n(\hat{t}_n, T^0) = \frac{15919}{21600} > 0,$$

that is, the small jurisdiction cannot increase its revenue by an upward deviation from $t = t^0$.

This finally concludes the demonstration that there is a regime-II Nash equilibrium of $(T_{II}(\pi^0), t_{II}(\pi^0))) = (T^0, t^0)$ for the parameter vector $\pi^0 := (3/5, 1, 3/2, 3/2, 8/5, 1)$.

B.5 Existence of Neighborhood Around π^0

The Jacobian of the two-equation system of first-order conditions characterizing the equilibrium is non-vanishing. The Jacobian in the region of a regime II equilibrium is given by,

$$\begin{aligned} J(T,t) &= \begin{bmatrix} \frac{\partial^2 R(T,t)}{\partial T^2} & \frac{\partial^2 R(T,t)}{\partial T \partial t} \\ \frac{\partial^2 r(t,T)}{\partial t \partial T} & \frac{\partial^2 r(t,T)}{\partial t^2} \end{bmatrix} \\ &= \begin{bmatrix} -\frac{4}{D} & \frac{2}{D} \\ \frac{2}{D} & -\frac{4}{D} \end{bmatrix}, \end{aligned} \tag{B.12}$$

which is of course, negative definite guaranteeing that it is valid to imply the Implicit Function Theorem around an equilibrium point.

B.6 Rescale by λ

Then, let $\pi(\lambda) = (h^0, H, \lambda d^0, \lambda D^0, \lambda E^0, S^0)$ denote a vector in which λ scales the transaction cost parameters, preserving their relative values. We now show how to construct a Nash equilibrium for any value of $\lambda > 0$ given the existence of an equilibrium with $\lambda = 1$, as established in the proceeding section. To do this, we need only note that when the equilibrium tax rates vary in proportion to λ, all of the other equilibrium conditions of the model continue to be satisfied. This can easily be seen because the base functions are homogeneous of degree zero in λ and both tax rates. The revenue functions are homogeneous of degree one in the tax rates and degree zero in λ. Then, all of the analysis for the case of $\lambda = 1$ can be reconstructed because the equilibrium conditions are still satisfied at tax rates that vary in proportion to λ. This obtains, uncountably many possible equilibrium tax rates.

B.7 Formal Statement of Existence

Thus, we can summarize:

Proposition 6. *Let $\pi(\lambda) = (h, H, \lambda d, \lambda D, \lambda E, S)$ denote a vector in which λ scales the transaction cost parameters, preserving their relative values. There exists a vector of parameter values $\pi^0 = (h^0, H^0, d^0, D^0, E^0, S^0) \in \mathbb{R}^6_{++}$ such that, for every $\lambda > 0$, there exists a unique regime-II Nash equilibrium $\forall\ \hat{\pi} = \hat{\pi}^0(\lambda)$ in which some but not all residents of the small jurisdiction buy the specialized good online and some but not all residents cross-border shop for the standardized good. Furthermore, for some number $\epsilon > 0$, and for every $\lambda > 0$, there exists a unique regime-II Nash equilibrium for all π such that $\|\pi - \pi^0(\lambda)\| < \epsilon$, i.e., for all points in the parameter space within an ϵ-ball around $\hat{\pi}^0(\lambda)$.*

Of course, in the paper we also characterize equilibria when transaction costs are sufficiently high (regime I) or sufficiently low (regime II). Formal mathematical proofs of existence in these cases can be derived. Interested readers should consult Agrawal and Wildasin (2019) for these formal deviations.

C Online Appendix C

The derivation of our comparative statics follows simply from differentiating the equilibrium tax rates. Alternatively, we can take the approach in Caputo (1996). Here, we do this for the general model. The first order conditions for an equilibrium where some but not all residents buy online are:

$$\begin{aligned} \frac{\partial R(T,t)}{\partial T} &= h + \frac{t-T}{d} + \frac{S(E+(t-T)S)}{D} - T(\frac{1}{d} + \frac{S^2}{D}) &= 0 \\ \frac{\partial r(t,T)}{\partial t} &= H - h + \frac{T-t}{d} + SH - \frac{S(E+(t-T)S)}{D} - t(\frac{1}{d} + \frac{S^2}{D}) &= 0 \end{aligned} \tag{C.1}$$

The Jacobian in the region of a regime II equilibrium is given by,

$$\begin{aligned} J(T,t) &= \begin{bmatrix} \frac{\partial^2 R(T,t)}{\partial T^2} & \frac{\partial^2 R(T,t)}{\partial T \partial t} \\ \frac{\partial^2 r(t,T)}{\partial t \partial T} & \frac{\partial^2 r(t,T)}{\partial t^2} \end{bmatrix} \\ &= \begin{bmatrix} -\frac{2}{d} - \frac{2S^2}{D} & \frac{1}{d} + \frac{S^2}{D} \\ \frac{1}{d} + \frac{S^2}{D} & -\frac{2}{d} - \frac{2S^2}{D} \end{bmatrix}. \end{aligned} \tag{C.2}$$

The determinant is

$$|J| = \frac{3(S^2 d + D)^2}{D^2 d^2}. \tag{C.3}$$

Differentiating the first order conditions with respect to E and following Cramer's Rule means the comparative statics are given by:

$$\begin{aligned} \frac{\partial T^{II}}{\partial E} &= -\frac{1}{|J|} \begin{vmatrix} \frac{S}{D} & \frac{1}{d} + \frac{S^2}{D} \\ -\frac{S}{D} & -\frac{2}{d} - \frac{2S^2}{D} \end{vmatrix} \\ &= \frac{1}{\frac{3(S^2d+D)^2}{D^2d^2}} \left(\frac{S(S^2 d + D)}{D^2 d} \right) \\ &= \frac{dS}{3(S^2 d + D)} \end{aligned} \tag{C.4}$$

and

$$\begin{aligned} \frac{\partial t^{II}}{\partial E} &= -\frac{1}{|J|} \begin{vmatrix} -\frac{2}{d} - \frac{2S^2}{D} & \frac{S}{D} \\ \frac{1}{d} + \frac{S^2}{D} & -\frac{S}{D} \end{vmatrix} \\ &= -\frac{1}{\frac{3(S^2d+D)^2}{D^2d^2}} \left(\frac{S(S^2 d + D)}{D^2 d} \right) \\ &= -\frac{dS}{3(S^2 d + D)} \end{aligned} \tag{C.5}$$

Notice that when evaluated at $d = D$ and $S = 1$, we obtain $\frac{\partial T^{II}}{\partial E} = \frac{1}{6}$ and $\frac{\partial t^{II}}{\partial E} = -\frac{1}{6}$, consistent with (21).

D Online Appendix D

The text presents various robustness checks and extensions to the model in section 6. In this appendix, we present the formal setup necessary to derive these results.

D.1 Section 6.1

In the presence of perfect audits, modifying the revenue functions yields

$$\begin{array}{rcl} R(T,t) & = & T(1-a)\left(h+\frac{t-T}{D}\right)+Tah \\ r(t,T) & = & (1-a)t\left(1-h+\frac{T-t}{D}\right)+ta(1-h), \end{array} \tag{D.1}$$

and the Nash equilibrium tax rates are

$$\begin{array}{rcl} T_N & = & \frac{1}{3}\frac{D}{1-a}(1+h) \\ t_N & = & \frac{1}{3}\frac{D}{1-a}(2-h), \end{array} \tag{D.2}$$

Notice that as $a \to 0$, the model converges to the canonical model (all cross-border shoppers pay the origin rate). As $a \to 1$ governments extract all the consumer surplus from residents because they have perfect enforcement at destination (effectively closing the border). Thus, destination taxation eliminates tax competition for Leviathan governments results in tax rates that extract all surplus. Of particular interest are the comparative statics with respect to the audit parameter, which are

$$\begin{array}{rcl} \frac{\partial T_N}{\partial a} & = & \frac{D(1+h)}{3(a-1)^2} > 0 \\ \frac{\partial t_N}{\partial a} & = & \frac{D(2-h)}{3(a-1)^2} > 0 \end{array}. \tag{D.3}$$

This clearly indicates that with revenue maximizing governments, enforcing destination taxation (even marginally) will raise tax rates in both jurisdictions as it reduces tax competition. All other results described in the text follow from these.

D.2 Section 6.2

In this section, we derive the tax rates in the presence of multiple agglomerations (assuming it exists, i.e., assuming that E and the consumer valuation of the commodity are of a size that does not push the jurisdiction to the past or future) of a regime II equilibrium. In regime II, the tax revenue functions are given by the specialized tax bases given in section 6.2 plus the standardized tax base

$$R(T,t) = T\left(h+\frac{t-T}{D}\right)+S\left(\frac{E+(t-T)S}{D}\right)+s\left(1-\frac{E+(T-t)s}{D}\right) \tag{D.4}$$

$$r(t,T) = t\left(1-h+\frac{T-t}{D}\right)+S\left(1-\frac{E+(t-T)S}{D}\right)+s\left(\frac{E+(T-t)s}{D}\right). \tag{D.5}$$

and the Nash equilibrium tax rates in this regime become

$$\begin{array}{rcl} T_{II} & = & \frac{D}{S^2+s^2+1}\left[\frac{2}{3}s+\frac{1}{3}\left(1+S+h+\frac{E}{D}(S-s)\right)\right] \\ t_{II} & = & \frac{D}{S^2+s^2+1}\left[\frac{2}{3}(S+1)-\frac{1}{3}\left(h-s+\frac{E}{D}(S-s)\right)\right]. \end{array} \tag{D.6}$$

(Notice, taking the limit as $s \to 0$ and $S \to 1$ yields the equation (18) in the text.) Hence, differentiating (D.6),

$$\begin{array}{rcl} \frac{\partial T_{II}}{\partial E} & = & \frac{1}{3}\frac{S-s}{S^2+s^2+1} \\ \frac{\partial t_{II}}{\partial E} & = & -\frac{1}{3}\frac{S-s}{S^2+s^2+1}. \end{array} \tag{D.7}$$

Given that revenues are $R_{II} = \frac{S^2+s^2+1}{D}T_{II}^2$ and $r_{II} = \frac{S^2+s^2+1}{D}t_{II}^2$, the comparative statics regarding tax revenues, discussed in proposition 5, also hold. Furthermore, the change in aggregate revenue remains the same as in proposition 5 if $S - s > 0$. Specifically,

$$\frac{\partial(R_{II}+r_{II})}{\partial E} = - \quad 2\frac{S^2+s^2+1}{D}(t_{II}-T_{II})\left(\frac{1}{3}\frac{S-s}{S^2+s^2+1}\right), \tag{D.8}$$

which is positive if $S - s > 0$ because this also implies $T_{II} > t_{II}$.

D.3 Section 6.3

To solve the model when both taxes bases may be taxed at different rates, we can solve the model using only the specialized tax base. When E/D takes on intermediate values, the tax revenue functions become

$$R(T,t) = TS\left(\frac{E+(t-T)S}{D}\right) \tag{D.9}$$

$$r(t,T) = tS - tS\left(\frac{E+(t-T)S}{D}\right). \tag{D.10}$$

and the Nash equilibrium tax rates on the specialized base in this regime (assuming it exists, i.e., assuming that E and the consumer valuation of the commodity are of a size that does not push the jurisdiction to the past or future) become

$$\begin{array}{rcl} T_{II} & = & D\left(\frac{1}{3}+\frac{1}{3}\frac{E}{D}\right) \\ t_{II} & = & D\left(\frac{2}{3}-\frac{1}{3}\frac{E}{D}\right), \end{array} \tag{D.11}$$

The tax differential is

$$T_{II} - t_{II} = D\left(\frac{2}{3}\frac{E}{D}-\frac{1}{3}\right). \tag{D.12}$$

As can be seen by the equilibrium comparative statics with respect to the cost of online shopping, the sign of the comparative statics remain the same as proposition 5. The equilibrium tax rate on the standardized base simply solve the canonical model and are still given by (13):

$$\begin{aligned} T = & \quad D\left(\frac{1}{3} + \frac{1}{3}h\right) \\ t = & \quad D\left(\frac{2}{3} - \frac{1}{3}h\right). \end{aligned} \tag{D.13}$$

Notice that the equilibrium tax base in (18) is simply a linear combination of the two bases under this partitioned problem (i.e., $\frac{2}{3} + \frac{1}{3}\left(h + \frac{E}{D}\right) = \left(\frac{1}{3} + \frac{1}{3}\frac{E}{D}\right) + \left(\frac{1}{3} + \frac{1}{3}h\right)$ for the large jurisdiction and similarly for the small jurisdiction). At the same time, in (18), the inverse of the tax base externality is half as large as in the partitioned problem (i.e., $D/2$ versus D).

D.4 Section 6.4

The equilibrium tax rates follow directly by extending the intuition of the partitioned problem in the prior case to many goods.

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