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Cash-Flow Business Taxation Revisited: Bankruptcy, Risk Aversion and Asymmetric Information

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Abstract

It is well-known that cash-flow business taxes with full loss-offset, and their present-value equivalents, are neutral with respect to firms' investment decisions when firms are risk-neutral. We study the effects of cash-flow business taxation when there is bankruptcy risk, when firms are risk-averse, and when financial intermediaries face asymmetric information problems in financing heterogeneous firms. Cash-flow taxes remain neutral under bankruptcy risk alone, but can distort the entry and investment decisions of firms under both risk-aversion and asymmetric information. The ACE tax distorts firms' decisions when bankruptcy risk exists. We characterize the nature of such distortions and consider how cash-flow taxes would have to be amended to achieve neutrality.

Key Words: cash-flow tax, risk-averse firms, asymmetric information
JEL: H21, H25

1 Introduction

A classic result in the design of business taxes due to Brown (1948) concerns the neutrality of cash-flow taxation. Investment decisions undertaken in a world of full certainty will be unaffected by a tax imposed on firms' cash flows, assuming there is full loss-offsetting (and the tax rate is constant, as shown by Sandmo, 1979). In effect, a cash-flow tax will divert a share of the pure profits or rents of the firm to the government. This is of obvious potential policy interest since it represents a non-distorting source of tax revenue.

Not surprisingly, the so-called Brown tax inspired a sizable literature on neutral business tax design, much of which generalized Brown's neutrality result to taxes that are equivalent to cash-flow taxes in present value terms. Boadway and Bruce (1984) show that the cash-flow tax is a special case of a more general class of neutral business taxes that have the property that the present value of deductions for future capital costs (interest plus depreciation) arising from any investment just equals initial investment expenditures. (Current costs are assumed to be fully deductible on a cash basis when incurred, though they can be capitalized as well with no difficulty.) They characterized a general neutral business tax satisfying this property as follows. Investment expenditures are added to a capital account each year, and each year the capital account is depreciated at a rate specified for tax purposes. Capital costs deducted from the tax base in each tax year consist of the cost of capital and the depreciation rate applied to the book value of the capital account. We refer to this as a Capital Account Allowance (CAA) tax. The CAA tax is neutral regardless of the depreciation rate used, as long as full loss-offsetting applies. Moreover, the depreciation rate used for tax purposes can be arbitrary and can vary from year to year. Its pattern can be chosen so that negative tax liabilities are mitigated. Indeed, the firm itself can choose the depreciation rate to use, possibly contingent on minimizing tax losses in any given year. In effect, the CAA tax system allows the firm to carry forward any unused deductions for investment at the interest rate.

More generally, neutrality can be achieved by a business tax in which the present value of future tax bases just equals the present value of cash flows. An example of a cash-flow equivalent tax system of this sort is the Resource Rent Tax (RRT) proposed by Garnaut and Clunies-Ross (1975) for the taxation of non-renewable natural resources. In their version, firms starting out are allowed to accumulate negative cash flows in an account that rises each year with the cost of capital. Once the account becomes positive, cash flows are taxed as they occur. Like the Brown tax or the CAA tax, the RRT is neutral with respect to decisions by the firm, including extraction in the case of resource firms. Negative cash flows are carried forward at the cost of capital thereby achieving the equivalent of cash-flow taxation.

These basic results continue to apply if returns to investment are uncertain, provided firms are risk-neutral. Fane (1987) shows that neutrality holds under uncertainty as long as tax credits and liabilities are carried forward at the risk-free nominal interest rate, and that tax credits and liabilities are necessarily redeemed eventually. Bond and Devereux (1995) show that the CAA tax remains neutral in the presence of uncertainty and the possibility of bankruptcy provided that a risk-free interest rate applied to the value of the capital account is used for the cost of capital deduction, that any unused negative tax credits are refunded in the event of bankruptcy, and that the valuation of risky assets satisfy the value additivity principle.[1] The use of a risk-free discount rate reflects the assumption that there is no risk to the firm associated with postponing capital deductions into the future (i.e., no political risk). Boadway and Keen (2015) show that the same neutrality result applies to the RRT in the presence of uncertainty. Bond and Devereux (2003) extend these results to the Allowance for Corporate Equity (ACE) tax, which is a version of the CAA tax that allows actual interest deductions alongside a cost of capital deduction for equity-financed investment. (They assume that there are no rents earned by bond-holders.) They also show that neutrality can be achieved using the more general case of cash-flow taxation proposed by Meade (1978), referred to as (R+F)-base cash-flow taxation, in which both real and financial cash flows are included in the base. Recently, these results have been extended to consider the effect of cash-flow taxation on the entry decision of firms/entrepreneurs. Kanniainen and Panteghini (2012) show, using an option-value model for determining entry (and exit) of entrepreneurs, that cash-flow taxation distorts the entry decision unless the cash-flow tax rate is same as the wage tax rate potential entrepreneurs face in alternative employment.

These neutrality results have been the inspiration for some well-known policy proposals, some of which have been implemented. A cash-flow business tax was recommended by the US Treasury (1977), Meade (1978) in the UK, and the President's Panel (2005) in the USA. The latter two both recommended additional cash-flow taxation to apply to financial institutions. At the same time, refundability of outstanding tax losses on firms that wind up was not proposed. The Australian Treasury (2010) (the Henry Report) recommended an RRT for the mining industries in Australia. Several bodies have recommended an ACE corporate tax system, including the Institute for Fiscal Studies (1991), the Mirrlees Review (2011) and Institut d'Economia de Barcelona (2013). Some instances of cash-flow-equivalent

[1] The value additivity principle implies that the present value of the sum of stochastic future payoffs is equal to the sum of the present values of these payoffs, and is consistent with a no-arbitrage principle in the valuation of assets.

taxes have been implemented, although often without perfect loss-offsetting. ACE taxes have been deployed in a number of countries, including Brazil, Italy, Croatia and Belgium. Reviews of their use may be found in Klemm (2007), de Mooij (2011), Panteghini, Parisi, and Pighetti (2012), and Princen (2012). Cash-flow-type taxes with full loss-offset are used in the Norwegian offshore petroleum industry (reviewed in Lund, 2014), and the RRT was applied temporarily in the Australian mining industry.

The neutrality of cash-flow taxation no longer applies when the simple assumptions are relaxed. Suppose firms' owners are risk-averse so that part of the return to investment is compensation for risk. A cash-flow tax applies to both rents and returns to risk-taking, and these two streams cannot be distinguished. As Domar and Musgrave (1944) famously show, risk-averse savers faced with a proportional tax on capital income with full loss-offset would be expected to increase the proportion of their portfolio held as risky assets, although the results become murkier when the proceeds from the tax are returned to savers by the government, as thoroughly discussed in Atkinson and Stiglitz (1980) and Buchholz and Konrad (2014). These results readily extend to an entrepreneurial firm, as shown in Mintz (1981). This non-neutrality is not necessarily a bad thing if the government is better able to diversify risk that private savers, but for our purposes the cash-flow tax is no longer a neutral tax on rents.

Additional problems arise if there are credit-market imperfections due to asymmetric information. This has been approached as a problem of hidden action or adverse selection, first studied by Stiglitz and Weiss (1981) and subsequently by de Meza and Webb (1987). The latter authors show that if banks cannot observe the probability of a firm going bankrupt, there will be excessive credit extended to entrepreneurs. These results have been generalized by Boadway and Keen (2006) and Boadway and Sato (2011) to allow for ex post monitoring by banks to verify that firms claiming bankruptcy are truly insolvent. Moral hazard in credit markets can also give rise to market failure, such as in Townsend (1979), Diamond (1984), Williamson (1986, 1987) and Bernanke, Gertler and Gilchrist (1996), where the costs of observing realized returns ex post for the lender generates an agency cost that tends to distort the market equilibrium. Parts of the literature on credit market failures resulting from asymmetric information are surveyed in Boadway and Tremblay (2005).

In this paper, we revisit the use of cash-flow taxation as a neutral rent-collecting device when firms might be risk-averse and asymmetries of information may exist in capital markets. We do so in a simple partial equilibrium model of risk-averse entrepreneurs who vary in their productivity, so that returns to infra-marginal entrepreneurs generates rents. We study the effects of cash-flow taxation on both the entry decision of potential entrepreneurs (the extensive margin) and on the decision as to how much to borrow and invest (the intensive margin). The object is both to uncover distortions a cash-flow tax may impose on these two margins and to indicate how in theory the cash-flow tax would have to be revised so as to re-establish neutrality. We begin with the base case where entrepreneurs with projects of differing productivity are risk-neutral, and where banks can observe entrepreneurs' types. Investment outcomes are uncertain and entrepreneurs face the possibility of bankruptcy, which banks can verify only by engaging in costly monitoring. We then extend the analysis to allow for risk-averse entrepreneurs and adverse selection. Conducting the analysis using entrepreneurial firms is for simplicity. The analysis could be readily extended to public corporations.

2 The Basic Model with Risk-Neutral Entrepreneurs

There is a population of potential entrepreneurs with an identical endowment of wealth but different productivities as entrepreneurs. We assume that there is a single period. That allows us to suppress the entrepreneurs' consumption-savings decision, which simplifies our analysis considerably by allowing us to focus on production decisions. At the beginning of the period, potential entrepreneurs decide whether to enter a risky industry and invest their wealth there. If they do not enter, they invest their wealth in a risk-free asset and consume the proceeds at the end of the period. Also for simplicity, we suppress their labor income: all income comes from profits they earn if they enter the risky sector, or their initial wealth if they do not. Adding labor income (as in Kanniainen and Panteghini (2012)) would make no substantial difference for our result on business taxation as we discuss later.

Entrepreneurs who enter the risky industry choose how much to borrow to leverage their own equity investment, which determines their capital stock. After investment has been undertaken, risk is resolved. Those with good outcomes earn profits for the entrepreneur. Those with bad outcomes go bankrupt. Their production goes to their creditors, which are risk-neutral competitive banks. There are thus two decisions made by potential entrepreneurs. First, they decide whether to enter, which we can think of as an extensive-margin decision, and second, they decide how much to borrow to expand their capital, which is an intensive-margin decision. To begin with, we assume that entrepreneurs are risk-neutral. The details of events are as follows.

There is a continuum of potential entrepreneurs all of whom are endowed with initial wealth E. For simplicity, we assume that the production function is linear in capital K. The average product of capital, denoted R, is constant, but differs across entrepreneurs, and is distributed over $[0, \overline{R}]$. The value of output is subject to risk, and the stochastic value of output of a type$-R$ entrepreneur is $\tilde{\varepsilon}RK$, where $\tilde{\varepsilon}$ is distributed uniformly over $[0, \varepsilon_{\max}]$, with density $g = 1/\varepsilon_{\max}$. Then, the expected value of ε is

$$\overline{\varepsilon} \equiv \mathbb{E}[\tilde{\varepsilon}] = \frac{\varepsilon_{\max}}{2} = \frac{1}{2g} \tag{1}$$

For simplicity we assume that the distribution of $\tilde{\varepsilon}$ is the same for all entrepreneurs, so they differ only by their productivity R. Capital is financed by the entrepreneur's own equity and debt, and depreciates at the proportional rate δ per period. Entrepreneurs who do not enter invest all their wealth in an asset with a risk-free rate of return ρ, so consume $(1+\rho)E$. Since all potential entrepreneurs have the same alternative income, those with the highest productivity as entrepreneurs will enter the entrepreneurial sector. Let $\widehat{R}$ denote the average product of the marginal entrepreneur.

Entrepreneurs who enter will invest all their wealth in the risky firm. Then, E will be the value of own equity, which is the same for all entrepreneurs. The type$-R$ entrepreneur who has entered borrows an amount $B(R)$ so his aggregate capital stock is $K(R) = E + B(R)$. Let $B(R) \equiv \phi(R)K(R)$, where $\phi(R)$ is the leverage rate. Then $K(R)$ can be written:

$$K(R) = \frac{E}{1 - \phi(R)} \tag{2}$$

We assume that there is a maximum value of the capital stock, such that $K(R) \leqslant \overline{K}$, and moreover that $E < \overline{K}$ so the entrepreneur's wealth is less than the maximum size of the capital stock. By (2), this implies that $0 \leqslant \phi(R) \leqslant 1 - E/\overline{K} < 1$. Since we assume that all the entrepreneur's wealth is invested, the minimum level of capital for entrepreneurs who enter is E. Allowing entrepreneurs to invest only part of their wealth would complicate the analysis slightly without adding any insight. The entrepreneur's capital stock is therefore in the range $K(R) \in [E, \overline{K}]$. The assumption of a maximal capital stock reflects the notion that after some point additional capital is non-productive. It is like a strong concavity assumption on the production function, which precludes extreme outcomes that would otherwise occur with linear production.

By the assumptions we make below, equilibrium analysis applies separately to entrepreneurs of each type. Accordingly, consider a representative type$-R$ entrepreneur and drop the indentifier R from all functions for simplicity. The entrepreneur's ex post after-tax profits (or return to own equity) is given by:

$$\widetilde{\Pi} = \tilde{\varepsilon}RK + (1 - \delta)K - (1 + r)B - \widetilde{T} \tag{3}$$

where $\widetilde{T}$ is the tax paid and r is the interest rate, so $(1 + r)B$ is the repayment of interest and principal on the borrowing B. Note that the interest rate r is specific to type R since as we assume later banks observe entrepreneurs' types. The term $(1-\delta)K$ is the value of capital remaining after production, given the depreciation rate δ.

The tax liability under cash-flow taxation, evaluated after $\tilde{\varepsilon}$ is revealed, is given by:

$$\widetilde{T} = \tau\big(\tilde{\varepsilon}RK - (1+\rho)K + (1-\delta)K\big) = \tau\big(\tilde{\varepsilon}RK - \rho K - \delta K\big) \tag{4}$$

where τ is the tax rate and, as noted above, ρ is the risk-free interest rate. The cash-flow tax base in the middle expression consists of three terms. The first is the revenue of the firm, $\tilde{\varepsilon}RK$. The second, $(1+\rho)K$, is the deduction for investment. Since this occurs at the beginning of the period, we assume that the tax savings from depreciation are carried over to the end of the period with interest at rate ρ. Finally, the cash-flow tax is levied on selling or winding-up the business assets, $(1-\delta)K$, at the end of the period. Using (3) and (4), ex post after-tax profits may be written:

$$\widetilde{\Pi} = (1-\tau)\Big(\tilde{\varepsilon}RK + (1-\delta)K\Big) - (1+r)B + \tau(1+\rho)K \tag{5}$$

Note that (4) implicitly assumes that the tax system allows full loss-offsetting.

Entrepreneurs are confronted with bankruptcy when ε is too low to meet debt repayment obligations, that is, when $\widetilde{\Pi} < 0$. This occurs for entrepreneurs with $\varepsilon < \hat{\varepsilon}$, where $\hat{\varepsilon}$, which is specific to type R, satisfies:

$$0 = (1-\tau)\Big(\hat{\varepsilon}RK + (1-\delta)K\Big) - (1+r)B + \tau(1+\rho)K \tag{6}$$

In what follows, we refer to $\hat{\varepsilon}$ as *bankruptcy risk*. The higher the value of $\hat{\varepsilon}$, the greater the chances of the entrepreneur going bankrupt. In the event of bankruptcy, the loan is not repaid, and the remaining after-tax profits $(1-\tau)\big(\tilde{\varepsilon}RK + (1-\delta)K\big) + \tau(1+\rho)K$ go to the creditor. Combining (5) and (6), we obtain:

$$\widetilde{\Pi} = (1-\tau)(\tilde{\varepsilon} - \hat{\varepsilon})RK \quad \text{for} \quad \tilde{\varepsilon} \geqslant \hat{\varepsilon} \tag{7}$$

Eq. (6) can be written, using $\phi = B/K$, as $0 = (1-\tau)\big(\hat{\varepsilon}R + (1-\delta)\big) - (1+r)\phi + \tau(1+\rho)$. For given R, the relationship between bankruptcy risk $\hat{\varepsilon}$ and leverage ϕ depends on how the lending rate r facing entrepreneurs is obtained. This is determined by a competitive banking sector. Assume that banks are risk-neutral and can observe R for each entrepreneur, but cannot observe $\tilde{\varepsilon}$. Thus, we assume there is no adverse selection since banks know entrepreneurs' types, but there is moral hazard: part of the risk of bankruptcy is borne by banks rather than entrepreneurs. Imperfection of the financial market due to asymmetric information is captured by an ex post verification or monitoring cost in the event a firm declares bankruptcy. Following the financial accelerator model of Bernanke *et al* (1999), we assume that the verification cost is proportional to ex post output so takes the form

$c\tilde{\varepsilon}RK$, for $\tilde{\varepsilon} \leqslant \hat{\varepsilon}$. This might reflect the fact that the verification cost includes the costs of seizing the firm's output in a default.[2] The expected total verification cost for a given type of entrepreneur is:

$$\int_0^{\hat{\varepsilon}} c\tilde{\varepsilon}RKg d\tilde{\varepsilon} = cRKg\frac{\hat{\varepsilon}^2}{2} \tag{8}$$

Thus, the expected verification or monitoring cost increases with bankruptcy risk, $\hat{\varepsilon}$.

Competition among banks ensures that expected profits earned from lending to the representative entrepreneur of each type are zero. We assume that banks will not go bankrupt, so they pay the risk-free interest rate ρ on their deposits. Assuming also that banks incur no costs of operation, zero-expected bank profits imply $(1+\rho)B = (1+r)B\int_{\hat{\varepsilon}}^{\varepsilon_{\max}} gd\tilde{\varepsilon} + \int_0^{\hat{\varepsilon}} \widetilde{\Pi} gd\tilde{\varepsilon} - cRKg\hat{\varepsilon}^2/2$, using the fact that debt is only repaid if $\tilde{\varepsilon} > \hat{\varepsilon}$. Substituting (5) and integrating, we obtain:

$$(1+\rho)B = (1+r)B\int_{\hat{\varepsilon}}^{\varepsilon_{\max}} gd\tilde{\varepsilon} + \int_0^{\hat{\varepsilon}} \Big((1-\tau)\big(\tilde{\varepsilon}RK + (1-\delta)K\big) + \tau(1+\rho)K\Big)gd\tilde{\varepsilon} - cRKg\frac{\hat{\varepsilon}^2}{2}$$

$$= (1+r)B(1-g\hat{\varepsilon}) + (1-\tau)RKg\frac{\hat{\varepsilon}^2}{2} + \big((1-\tau)(1-\delta) + \tau(1+\rho)\big)Kg\hat{\varepsilon} - cRKg\frac{\hat{\varepsilon}^2}{2} \tag{9}$$

Eq. (9) reflects the fact that for $\varepsilon \leqslant \hat{\varepsilon}$, the banks retain after-tax profits, but face the verification costs $c\tilde{\varepsilon}RK$. This zero-profit condition applies for each type of entrepreneur, so in general each faces a different interest rate.

Note that the banks pay no cash-flow tax on their own profits, although they incur whatever taxes are owing (positive or negative) on the profits they receive on bankrupt projects. We assume initially that the bank pays the same tax as the firm would have been liable for had it remained in business. Later we consider the possibility that the bank is able to deduct the monitoring costs it incurs to verify that the firm actually is bankrupt. The absence of cash-flow taxation on banks is innocuous given our assumption that banks earn no pure profits or rents and are risk-neutral. Expected cash flows, and therefore expected tax liabilities, would be zero. In a world where banks earn rents, the cash-flow tax could be extended to them, as proposed initially by the Meade Report and discussed in the Mirrlees Review.

Using the zero-profit condition (6), interest and principal repayments $(1+r)B$ can be

[2] Bernanke and Gertler (1989) introduced a fixed verification cost in a business cycle model where there is asymmetric information between lenders and borrowers about the realized return on risky projects, while Townsend (1979) explored the design of debt contracts with verification costs that could either be fixed or functions of realized project output. See also Bernanke *et al* (1996) for an analysis of the implications of agency costs in lending contracts arising from asymmetric information about project outcome.

eliminated from (9) to give the following lemma.[3]

Lemma 1 *The leverage rate* $\phi \equiv B/K$*, for* $0 < \phi < 1 - E/\overline{K}$*, is given by:*

$$\phi(\hat{\varepsilon}, R, \tau, c) = \frac{1-\tau}{1+\rho}\Big(1 - \frac{g\hat{\varepsilon}}{2}\Big)\hat{\varepsilon}R + \frac{1-\tau}{1+\rho}(1-\delta) + \tau - \frac{cRg\hat{\varepsilon}^2}{2(1+\rho)} \tag{10}$$

We assume that entrepreneurs recognize how leverage affects the interest rate in (9). This is taken into account by (10). While in practise, entrepreneurs choose leverage ϕ, it is convenient for us to assume in our analysis that they choose bankruptcy risk $\hat{\varepsilon}$, which is related to leverage via (10). Routine differentiation of (10) gives properties of $\phi(\hat{\varepsilon}, R, \tau, c)$ that are useful in what follows:

$$\phi_{\hat{\varepsilon}} = (1-\tau)(1-g\hat{\varepsilon})\frac{R}{1+\rho} - \frac{cRg}{1+\rho}\hat{\varepsilon}; \quad \phi_c = -\frac{Rg}{2(1+\rho)}\hat{\varepsilon}^2; \quad \phi_R = \frac{1-\tau}{1+\rho}\Big(1 - \frac{g\hat{\varepsilon}}{2}\Big)\hat{\varepsilon} - \frac{cg}{2(1+\rho)}\hat{\varepsilon}^2;$$

$$\phi_\tau = -\Big(1 - \frac{g\hat{\varepsilon}}{2}\Big)\frac{\hat{\varepsilon}R}{1+\rho} - \frac{1-\delta}{1+\rho} + 1 = \frac{1-\phi}{1-\tau} - \frac{cRg\hat{\varepsilon}^2}{2(1-\tau)(1+\rho)} \tag{11}$$

$$\phi_{\hat{\varepsilon}\hat{\varepsilon}} = -(1-\tau+c)\frac{Rg}{1+\rho} \ < \ 0; \quad \phi_{\hat{\varepsilon}\tau} = -(1-g\hat{\varepsilon})\frac{R}{1+\rho} \ < \ 0$$

Prior to ε being revealed to the representative entrepreneur, expected profits accruing to the entrepreneur are $\overline{\Pi} \equiv \int_{\hat{\varepsilon}}^{\varepsilon_{\max}} \widetilde{\Pi} g d\tilde{\varepsilon}$. (Recall that for $\tilde{\varepsilon} < \hat{\varepsilon}$, profits are claimed by the bank.) Given the expression for $\widetilde{\Pi}$ in (5), this becomes:

$$\overline{\Pi} = \int_{\hat{\varepsilon}}^{\varepsilon_{\max}} \Big((1-\tau)\big(\tilde{\varepsilon}RK + (1-\delta)K\big) - (1+r)B + \tau(1+\rho)K\Big) g d\tilde{\varepsilon} \tag{12}$$

Using (9), (1) and (2) along with $B = K - E$, this may be written:

$$\overline{\Pi} = \left(\frac{1}{1-\phi(\hat{\varepsilon}, R, \tau, c)}\Big((1-\tau)(\bar{\varepsilon}R - \delta - \rho) - \frac{cRg\hat{\varepsilon}^2}{2}\Big) + 1 + \rho\right) E \equiv \overline{\pi}(\hat{\varepsilon}, R, \tau, c)E \tag{13}$$

where $\overline{\pi}(\hat{\varepsilon}, R, \tau, c)$ is expected profit per unit of own equity, $\overline{\Pi}/E$. Using (10), $\overline{\pi}(\hat{\varepsilon}, R, \tau, c)$ satisfies the following lemma.[4]

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[3]Proof: Using (6), the righthand side of (9) can be written:

$$\big((1-\tau)(\hat{\varepsilon}R + (1-\delta)) + \tau(1+\rho)\big)K(1-g\hat{\varepsilon}) + (1-\tau)RK\frac{g\hat{\varepsilon}^2}{2} + \big((1-\tau)(1-\delta) + \tau(1+\rho)\big)Kg\hat{\varepsilon} - \frac{cRg\hat{\varepsilon}^2}{2}$$

$$= (1-\tau)\Big(\hat{\varepsilon}(1-g\hat{\varepsilon}) + \frac{g\hat{\varepsilon}^2}{2}\Big)RK + \big((1-\tau)(1-\delta) + \tau(1+\rho)\big)K - \frac{cRg\hat{\varepsilon}^2}{2}$$

$$= (1-\tau)\Big(\hat{\varepsilon} - \frac{g\hat{\varepsilon}^2}{2}\Big)RK + \big((1-\tau)(1-\delta) + \tau(1+\rho)\big)K - \frac{cRg\hat{\varepsilon}^2}{2} \qquad \blacksquare$$

[4]Proof: Rewrite $\overline{\pi}$ in (13) as

$$\overline{\pi}(\hat{\varepsilon}, R, \tau, c) = \frac{1}{1-\phi}\Big((1-\tau)(R\bar{\varepsilon} - \delta - \rho) - \frac{cRg\hat{\varepsilon}^2}{2} + (1+\rho)(1-\phi)\Big)$$

Lemma 2

$$\overline{\pi}(\hat{\varepsilon}, R, \tau, c) = \frac{1-\tau}{1-\phi(\cdot)} \frac{Rg}{2} (\varepsilon_{\max} - \hat{\varepsilon})^2 \tag{14}$$

For future use, differentiate $\overline{\pi}(\cdot)$ in (13) with respect to $\hat{\varepsilon}$ to obtain:

$$\overline{\pi}_{\hat{\varepsilon}} = \frac{\Delta(\hat{\varepsilon}, R, \tau, c)}{1-\phi(\cdot)} \Big((1-\tau)(\overline{\varepsilon}R - \delta - \rho) - \frac{cRg\hat{\varepsilon}^2}{2} \Big) - \frac{c\hat{\varepsilon}Rg}{1-\phi(\cdot)} \tag{15}$$

where

$$\Delta(\hat{\varepsilon}, R, \tau, c) \equiv \frac{\phi_{\hat{\varepsilon}}(\hat{\varepsilon}, R, \tau, c)}{1-\phi(\hat{\varepsilon}, R, \tau, c)} \tag{16}$$

Differentiating with respect to τ and using the properties of $\phi(\hat{\varepsilon}, R, \tau, c)$ in (11), we obtain:[5]

$$\Delta_\tau = -\frac{c\hat{\varepsilon}Rg}{(1+\rho)(1-\tau)(1-\phi(\cdot))} \Big(1 + \frac{\phi_{\hat{\varepsilon}}(\cdot)}{1-\phi(\cdot)} \frac{\hat{\varepsilon}}{2} \Big) \tag{17}$$

Finally, since we assume risk-neutrality in this base case, the expected utility of entrepreneurs is given by $\overline{\Pi} = \int_{\hat{\varepsilon}}^{\varepsilon_{\max}} \widetilde{\Pi} g d\tilde{\varepsilon}$. Using (7), (2) and (14), this becomes:

$$\overline{\Pi} = (1-\tau)RK \int_{\hat{\varepsilon}}^{\varepsilon_{\max}} (\tilde{\varepsilon} - \hat{\varepsilon}) g d\tilde{\varepsilon} = \frac{(1-\tau)}{1-\phi(\cdot)} \frac{Rg}{2} (\varepsilon_{\max} - \hat{\varepsilon})^2 E = \overline{\pi}(\hat{\varepsilon}, R, \tau, c) E \tag{18}$$

3 Behavior of Entrepreneurs

Recall that entrepreneurs make two choices in sequence. First, they decide whether to undertake risky investments, given their productivity R. This is the extensive-margin decision. Then, if they enter, they decide how much to borrow to acquire more capital over and above their own equity, E. This is their intensive-margin decision. Once their shock $\tilde{\varepsilon}$ is revealed, their after-tax profits and therefore ex post utility are determined. We consider the intensive and extensive decisions in reverse order for a representative entrepreneur of a given type, and continue to suppress the type identifier R for simplicity.

From (10), we obtain $(1+\rho)(1-\phi) = -(1-t)(1-g\hat{\varepsilon}/2)\hat{\varepsilon}R + (1-t)(\rho+\delta) + cRg\hat{\varepsilon}^2/2$. Substituting this in the expression for $\overline{\pi}$ gives, using $\overline{\varepsilon} = \varepsilon_{\max}/2$ and $\varepsilon_{\max} = 1/g$:

$$\overline{\pi}(\hat{\varepsilon}, R, \tau, c) = \frac{1-\tau}{1-\phi} R\Big(\overline{\varepsilon} - \hat{\varepsilon}\Big(1 - \frac{g}{2}\hat{\varepsilon}\Big)\Big) = \frac{1-\tau}{1-\phi} \frac{Rg}{2} (\hat{\varepsilon} - \varepsilon_{\max})^2 \quad \blacksquare$$

[5]Proof: Differentiating $\Delta(\hat{\varepsilon}, \tau, c)$, we have $\Delta_\tau = \phi_{\hat{\varepsilon}\tau}/(1-\phi) + \phi_\tau \phi_{\hat{\varepsilon}}/(1-\phi)^2$. Using (11) for ϕ_τ and $\phi_{\hat{\varepsilon}\tau}$,

$$\Delta_\tau = -\frac{1}{1-\phi} \frac{R}{1+\rho} (1 - g\hat{\varepsilon}) + \frac{\phi_{\hat{\varepsilon}}}{1-\phi} \frac{1}{1-\tau} - \frac{\phi_{\hat{\varepsilon}}}{(1-\phi)^2} \frac{1}{1-\tau} \frac{cRg\hat{\varepsilon}^2}{2(1+\rho)}$$

Using (11) for $\phi_{\hat{\varepsilon}}$,

$$\Delta_\tau = -\frac{1}{1-\phi} \frac{1}{1-\tau} \frac{cRg\hat{\varepsilon}}{1+\rho} - \frac{\phi_{\hat{\varepsilon}}}{(1-\phi)^2} \frac{1}{1-\tau} \frac{cRg\hat{\varepsilon}^2}{2(1+\rho)} \quad \blacksquare$$

3.1 Choice of leverage: intensive margin

As mentioned, given (10) the choice of leverage ϕ is essentially the same as the choice of $\hat{\varepsilon}$, which determines $K = E/(1-\phi(\cdot))$. This follows because, even though ϕ is not necessarily monotonic in $\hat{\varepsilon}$, $\phi_{\hat{\varepsilon}\hat{\varepsilon}} < 0$ by (11). Differentiating (18) with respect to $\hat{\varepsilon}$ and using (15), we obtain:

$$\frac{d\overline{\Pi}}{d\hat{\varepsilon}} = \overline{\pi}_{\hat{\varepsilon}} E = \left(\frac{\Delta(\hat{\varepsilon}, R, \tau, c)}{1-\phi(\cdot)} \left((1-\tau)(\overline{\varepsilon} R - \delta - \rho) - \frac{cRg\hat{\varepsilon}^2}{2} \right) - \frac{c\hat{\varepsilon}Rg}{1-\phi(\cdot)} \right) E \tag{19}$$

where $\Delta(\hat{\varepsilon}, R, \tau, c)$ is defined in (16). Alternatively, differentiating $\overline{\pi}$ in (14),

$$\frac{d\overline{\Pi}}{d\hat{\varepsilon}} = \overline{\pi}_{\hat{\varepsilon}} E = \left(\Delta(\hat{\varepsilon}, R, \tau, c) - \frac{2}{\varepsilon_{\max} - \hat{\varepsilon}} \right) \frac{(1-\tau)Rg}{1-\phi} (\varepsilon_{\max} - \hat{\varepsilon})^2 E \tag{20}$$

Let $\hat{\varepsilon}^*$ be the optimal choice of $\hat{\varepsilon}$. If it is in the interior, to which we return below, $d\overline{\Pi}/d\hat{\varepsilon} = 0$ and we have from (20) that $\overline{\pi}_{\hat{\varepsilon}} = 0$ and

$$\Delta(\hat{\varepsilon}^*, R, \tau, c) = \frac{2}{\varepsilon_{\max} - \hat{\varepsilon}^*} > 0 \tag{21}$$

From (21) we obtain the following lemma.[6]

Lemma 3 *Assume $\hat{\varepsilon}^*$ is in the interior. Then,*

$$\frac{d\hat{\varepsilon}^*}{d\tau} \leqslant 0 \quad \text{as} \quad c \geqslant 0$$

Alternatively, $\hat{\varepsilon}^*$ may take on corner solutions at the top or bottom. From (10), $\hat{\varepsilon}^*$ takes on a minimum value of $\hat{\varepsilon}^* = 0$ when $\phi \leqslant \phi(0, R, \tau, c) = (1-\tau)(1-\delta)/(1+\rho) + \tau > 1$. The maximum value of $\hat{\varepsilon}^*$ satisfies $\phi(\hat{\varepsilon}, R, \tau, c) = 1 - E/\overline{K}$, which is assumed to be smaller than $\varepsilon_{\max}$ for any entrepreneur type.

To see how the level of R affects leverage and bankruptcy risk, assume again that $\hat{\varepsilon}^*$ is in the interior so $d\overline{\Pi}/d\hat{\varepsilon} = 0$ in (20). If the second-order conditions are satisfied, we have:

$$\frac{d^2\overline{\Pi}}{d\hat{\varepsilon}^2} \ \propto \ \Delta_{\hat{\varepsilon}}(\hat{\varepsilon}, R, \tau, c) - \frac{2}{(\varepsilon_{\max} - \hat{\varepsilon}})^2 \ < \ 0$$

[6] Proof: Differentiate the first-order condition (21) to obtain:

$$\Delta_\tau d\tau + \left(\Delta_{\hat{\varepsilon}} - \frac{2}{(\varepsilon_{\max} - \hat{\varepsilon}^*)^2} \right) d\hat{\varepsilon} = 0$$

The coefficient of $d\hat{\varepsilon}$ is negative by the second-order condition. Since $\Delta > 0$ by (21), $\phi_{\hat{\varepsilon}} > 0$ by the definition of Δ in (16). Lemma 3 follows since $\Delta_\tau \leqslant 0$ as $c \geqslant 0$ by (17) when $\phi_{\hat{\varepsilon}} > 0$.

Rewrite elements of (11) as

$$\phi_{\hat{\varepsilon}} = \Big(\frac{1-\tau}{1+\rho}(1-g\hat{\varepsilon}) - \frac{cg}{1+\rho}\hat{\varepsilon}\Big)R, \quad \phi_R = \Big(\frac{1-\tau}{1+\rho}(2-g\hat{\varepsilon}) - \frac{cg}{1+\rho}\hat{\varepsilon}\Big)\frac{\hat{\varepsilon}}{2}$$

From this, we see that if $\phi_{\hat{\varepsilon}} > 0$, then $\phi_R > 0$ and $\phi_{\hat{\varepsilon}R} > 0$. The following lemma then applies.[7]

Lemma 4 *Assuming that $\phi_{\hat{\varepsilon}} > 0$ at $\hat{\varepsilon}^*$ and the second-order conditions are satisfied,*

$$\frac{d\hat{\varepsilon}^*}{dR} \; > \; 0 \tag{22}$$

This lemma implies that the probability of bankruptcy is increasing in the productivity R of the entrepreneur.

3.2 Decision to undertake risky investment: extensive margin

Ex ante, entrepreneurs decide whether to undertake the risky investment or to opt for the risk-free option. In the risk-free option, they invest their wealth E at a risk-free return ρ, leading to consumption of $(1+\rho)E$. They enter if their expected income as an entrepreneur, given by $\overline{\Pi}$ in (13) or (18), is at least as great as their certain income if they invest their wealth in a safe asset and obtain consumption of $(1+\rho)E$, that is,

$$\overline{\Pi} = \overline{\pi}(\hat{\varepsilon}, R, \tau, c)E \geqslant (1+\rho)E \quad \text{or} \quad \overline{\pi}(\hat{\varepsilon}, R, \tau, c) \geqslant 1+\rho \tag{23}$$

From (18), $\overline{\Pi}$ is increasing in R. Given that $\hat{\varepsilon}$ is being optimized, the cutoff value of R, denoted $\widehat{R}$, will be uniquely determined by $\overline{\pi}(\hat{\varepsilon}, \widehat{R}, \tau, c) = 1+\rho$. Using the expression for $\overline{\pi}$ in (13), the following lemma is apparent.

Lemma 5 *The cutoff value of R is determined by:*

$$(1-\tau)(\overline{\varepsilon}\widehat{R} - \delta - \rho) - \frac{c\widehat{R}g\hat{\varepsilon}^2}{2} = 0 \tag{24}$$

Entrepreneurs with $R > \widehat{R}$ enter the risky sector and earn a rent. Those with $R < \widehat{R}$ invest their wealth in a risk-free asset, so earn no rent. The issue is to what extent does the cash-flow tax system serve as a tax on rents. To study this, consider first the social optimum as a benchmark.

[7] Proof: Differentiate $d\overline{\Pi}/d\hat{\varepsilon} = 0$ in (20), and use $\Delta_R = \phi_{\hat{\varepsilon}R}/(1-\phi) + \phi_{\hat{\varepsilon}}\phi_R/(1-\phi)^2 > 0$ and the second order-conditions on $\hat{\varepsilon}$.

4 The Constrained Social Optimum

For purposes of comparison with the outcome achieved under cash-flow taxation, it is useful to assume that the government confronts the same asymmetric information constraint that the banks do. That is, they must monitor ex post incomes of entrepreneurs who declare bankruptcy and incur the same cost as the banks. The constrained social optimum maximizes the surplus net of monitoring costs of entrepreneurs who invest in the risky sector since no surplus is generated either by the banks, which earn zero expected profits, or by potential entrepreneurs who invest in the safe outcome and earn $(1+\rho)E$. Social surplus denoted S can be defined as the expected value of production by entrepreneurs less the opportunity cost of financing the entrepreneurs' capital less expected monitoring costs. Financing costs include the cost of both debt and equity finance, $(1+\rho)B + (1+\rho)E = (1+\rho)K$.

For the representative entrepreneur with given R, S can be written:

$$S = \int_0^{\varepsilon_{\max}} \big(\tilde{\varepsilon}RK + (1-\delta)K\big) g d\tilde{\varepsilon} - (1+\rho)K - cRKg\frac{\hat{\varepsilon}^2}{2}$$

$$= \overline{\varepsilon}RK + (1-\delta)K - (1+\rho)K - cRKg\frac{\hat{\varepsilon}^2}{2} \tag{25}$$

where $\hat{\varepsilon}$ is determined by (6) with $\tau = 0$, or equivalently from (10),

$$\phi(\hat{\varepsilon}, R, 0, c) = \Big(1 - \frac{g\hat{\varepsilon}}{2}\Big)\frac{\hat{\varepsilon}R}{1+\rho} + \frac{1-\delta}{1+\rho} - \frac{cRg\hat{\varepsilon}^2}{2(1+\rho)} \tag{26}$$

Note that S includes the surplus earned by the investments of entrepreneurs who go bankrupt since this accrues to the banks, less ex post verification costs. This expression for S applies whether taxes are in place or not.

We can show that if there are no taxes, entrepreneurial expected profit maximization leads to the constrained social optimum. That is, there is no market failure. To see this, note that with no taxes, expected profits in (12) become:

$$\overline{\Pi} = \int_{\hat{\varepsilon}}^{\varepsilon_{\max}} \Big(\tilde{\varepsilon}RK + (1-\delta)K - (1+r)B\Big) g d\tilde{\varepsilon} \tag{27}$$

and the banks' zero profits expression (9) reduces to:

$$(1+\rho)B = (1+r)B\int_{\hat{\varepsilon}}^{\varepsilon_{\max}} g d\tilde{\varepsilon} + \int_0^{\hat{\varepsilon}} \big(\tilde{\varepsilon}RK + (1-\delta)K\big) g d\tilde{\varepsilon} - cRKg\frac{\hat{\varepsilon}^2}{2} \tag{28}$$

Adding (27) and (28), we obtain:

$$\overline{\Pi} - (1+\rho)E = \int_0^{\varepsilon_{\max}} \big(\tilde{\varepsilon}RK + (1-\delta)K - (1+\rho)(B+E)\big) g d\tilde{\varepsilon} - cRKg\frac{\hat{\varepsilon}^2}{2} = S \tag{29}$$

Therefore, if the entrepreneur maximizes expected profits $\overline{\Pi}$, social surplus S will be maximized as well since $(1+\rho)E$ is constant.

To verify this in another way, rewrite S as follows using $K = E + B = E/(1-\phi)$:

$$S = \Big(\overline{\varepsilon}R - \delta - \rho - \frac{cRg\hat{\varepsilon}^2}{2}\Big)\frac{E}{1-\phi(\cdot)}$$

where $\phi(\cdot)$ is given by (26). Differentiating with respect to $\hat{\varepsilon}$ yields:

$$\frac{dS}{d\hat{\varepsilon}} = \frac{\phi_{\hat{\varepsilon}}}{(1-\phi)^2}\Big(\overline{\varepsilon}R - \delta - \rho - \frac{cRg\hat{\varepsilon}^2}{2}\Big)E - \frac{cRg\hat{\varepsilon}}{1-\phi}E = \frac{d\overline{\Pi}}{d\hat{\varepsilon}} \tag{30}$$

where the second equality follows from $d\overline{\Pi}/d\hat{\varepsilon} = \overline{\pi}_{\hat{\varepsilon}}E$, with $\overline{\pi}_{\hat{\varepsilon}}$ given by (15) when $\tau = 0$. This result obviously applies for all entrepreneurial types. Therefore, the choice of leverage by entrepreneurs is efficient in the no-tax case.

Consider now the extensive-margin decision. Entry by a type$-R$ entrepreneur will be efficient if $S > 0$. Let R^S be the marginal entrepreneur-type in the social optimum. For this entrepreneur, $S = 0$, or using (25),

$$\overline{\varepsilon}R^S - \delta - \rho - cR^S g\frac{\hat{\varepsilon}^2}{2} = 0 \tag{31}$$

This corresponds with (24) when $\tau = 0$, so $\widehat{R} = R^S$ and entry is efficient as well.

When $\tau > 0$, following the same procedure of combining (12) with (9), we obtain:

$$\overline{\Pi} - (1+\rho)E = \int_{\hat{\varepsilon}}^{\varepsilon_{\max}} \Big((1-\tau)\big(\tilde{\varepsilon}RK + (1-\delta)K\big) - (1+\rho)(B+E)\Big)gd\tilde{\varepsilon} - cRKg\frac{\hat{\varepsilon}^2}{2} = S - \overline{T}$$

where $\overline{T}$ is expected tax revenue. Now, maximizing entrepreneurs' net surplus $\overline{\Pi} - (1+\rho)E$ will generally not be equivalent to maximizing social surplus S. We turn to this issue next.

5 Cash-Flow Taxation with Risk-Neutral Entrepreneurs

The base-case model discussed in the previous section includes both bankruptcy, when entrepreneurs are unable to repay their loans fully, and asymmetric information, in the sense that banks can only verify bankruptcy with costly ex post monitoring. In this section, we consider first the effect of cash-flow taxation in the base-case economy, in particular whether such a tax is neutral. We then consider two special cases. In the first, the banks are fully informed in the sense that they know whether firms are bankrupt. This is the case where monitoring is costless, so $c = 0$. It corresponds with Bond and Devereux (1995) where entrepreneurs are risk-neutral and there is no asymmetric information, although here we allow for limited liability in the event of bankruptcy. In the second, we let monitoring costs c incurred by the banks be deductible from the tax base of bankrupt firms.

The intensive- and extensive-margin choices of entrepreneurs in the base case are guided by (19) and (23). Consider first the marginal entrepreneur $\widehat{R}$ for whom (24) applies. Substituting this into (19), we obtain:

$$\frac{d\overline{\Pi}}{d\hat{\varepsilon}} = -\frac{c\widehat{R}g\hat{\varepsilon}}{1-\phi}E < 0$$

This implies that $\hat{\varepsilon} = 0$ for $R = \widehat{R}$, so $\widehat{R}$ satisfies

$$\overline{\varepsilon}\widehat{R} - \delta = \rho \tag{32}$$

and is independent of τ. Thus, the extensive-margin decision is not affected by a cash-flow tax, and the marginal entrepreneur takes no bankruptcy risk.

The leverage decision for an infra-marginal type$-R$ entrepreneur is governed by (19), where $\hat{\varepsilon}$ will be in the interior if it satisfies $d\overline{\Pi}/d\hat{\varepsilon} = 0$. If $\hat{\varepsilon}^*$ is in the interior, differentiating $\phi(\cdot)$ gives:

$$\frac{d\phi}{d\tau} = \phi_{\hat{\varepsilon}}\frac{d\hat{\varepsilon}}{d\tau} + \phi_{\tau} \tag{33}$$

By Lemma 3, $d\hat{\varepsilon}^*/d\tau < 0$ when $c > 0$ and $\hat{\varepsilon}^* > 0$, and by (16) and (21), $\phi_{\hat{\varepsilon}} > 0$, while $\phi_{\tau} \gtreqless 0$ by (11). Therefore, $d\phi/d\tau$ is ambiguous in sign, but in general cash-flow taxation is distortionary.

Further insight into entrepreneurs' decisions can be obtained by considering the two special cases mentioned above.

5.1 Costless monitoring (c = 0)

This corresponds with the case studied by Bond and Devereux (1995) where there is no asymmetric information. Consider first the extensive-margin decision, where (24) applies. When $c = 0$, $\widehat{R}$ satisfies (32) which is independent of τ. Equivalently, $\overline{\pi} \geqslant 1 + \rho$ by (23), so for the marginal entrepreneur, $\overline{\pi} = 1 + \rho$. Eq. (32) then follows from (13). Thus, the cash-flow tax does not distort the entry decision in this case. The value of $\widehat{R}$ that satisfies (32) is the value R^S that satisfies (31) in the constrained social optimum.

Next, consider the effect of the cash-flow tax on leverage. With $c = 0$, (19) becomes:

$$\frac{d\overline{\Pi}}{d\hat{\varepsilon}} = \overline{\pi}_{\hat{\varepsilon}}E = \frac{\Delta(\hat{\varepsilon}, R, \tau, c)}{1-\phi(\cdot)}(1-\tau)(\overline{\varepsilon}R - \delta - \rho)E$$

For the marginal entrepreneur, (32) implies that $\overline{\pi}_{\hat{\varepsilon}} = 0$, so $d\overline{\Pi}/d\hat{\varepsilon}|_{R=\widehat{R}} = 0$. Therefore, leverage ϕ and thus K are indeterminate for the marginal entrepreneur and independent of

τ. For inframarginal entrepreneurs, $\bar{\varepsilon}R - \delta - \rho > 0$ since $R > \widehat{R}$, so $\pi_{\hat{\varepsilon}}$ has the same sign as $\Delta(\cdot) = \phi_{\hat{\varepsilon}}/(1-\phi)$. This is positive given that $\phi_{\hat{\varepsilon}} > 0$ when $c = 0$ by (11). Therefore, $\hat{\varepsilon}$ takes its maximum value with $\phi(\hat{\varepsilon}^*, R, \tau, c) = 1 - E/\overline{K}$. Since $\phi_{\hat{\varepsilon}} > 0$ and $\phi_\tau > 0$ by (11), we have that $d\hat{\varepsilon}^*/d\tau < 0$ to keep ϕ constant. While $\hat{\varepsilon}^*$ changes, τ does not distort ϕ or the capital stock $K = \overline{K} = E/(1-\phi)$.

Note that differentiating (13) with $c = 0$ and given that ϕ is fixed at its maximum value, we obtain

$$\frac{d\overline{\Pi}}{d\tau} = -\frac{\bar{\varepsilon}R - \delta - \rho}{1-\phi}E \; < \; 0 \text{ for } R > \widehat{R}$$

That is, the cash-flow tax decreases after-tax expected profits or rents for all inframarginal firms without changing their behavior. Moreover, the reduction is increasing in productivity R. Marginal firms are not affected since their expected profits are zero.

We can verify that this is the same choice of capital as in the constrained social optimum. From (25), when $c = 0$ social surplus for a type$-R$ firm becomes $S = (\bar{\varepsilon}R - \rho - \delta)K$. For the marginal entrepreneur, social surplus is zero and independent of K by (32), so the latter is indeterminate and remains so under cash-flow taxation. For inframarginal entrepreneurs, S is increasing in K, so they all borrow and invest such that $K = \overline{K}$, just as we have found under cash-flow taxation.

In summary, when $c = 0$, τ distorts neither ϕ nor $\widehat{R}$. This corresponds with the standard case of cash-flow tax neutrality when entrepreneurs are risk-neutral and capital markets are perfect. The tax applies to rents, and there are no rents for the marginal entrepreneur. Note that the CAA and RRT tax systems will also be neutral provided the rate at which unused capital deductions or cash flows are carried forward is the risk-free interest rate ρ.

5.2 Monitoring costs deductible

We have assumed in the base case that monitoring costs cannot be deducted against the taxable income of bankrupt firms. We can readily see that this is a potential source of inefficiency of cash-flow taxation by comparing expected profits with social surplus. Combining (12) with (9) as before and using $B + E = K$, we obtain after simplification:

$$\overline{\Pi} - (1+\rho)E = (1-\tau)S - \tau cRKg\frac{\hat{\varepsilon}^2}{2} \tag{34}$$

If the banks were allowed to deduct monitoring costs, a tax benefit would be created equal to the last term in (34). In that case, (34) would reduce to $\overline{\Pi} - (1+\rho)E = (1-\tau)S$, and we might think that maximizing after-tax expected profits $\overline{\Pi}$ would be equivalent to maximizing S. However, matters are not so simple.

Suppose we allow bankrupt firms to deduct monitoring costs $c\tilde{\varepsilon}RK$, although this may be very difficult to do on a firm-by-firm basis from a tax compliance point of view. Where c appears in the basic model, $(1-\tau)c$ would appear instead. Eq. (10) and its relevant properties would become:

$$\phi(\hat{\varepsilon}, R, \tau, c) = \frac{1-\tau}{1+\rho}\Big(1 - \frac{g\hat{\varepsilon}}{2}\Big)\hat{\varepsilon}R + \frac{1-\tau}{1+\rho}(1-\delta) + \tau - \frac{(1-\tau)cRg\hat{\varepsilon}^2}{2(1+\rho)}$$

where

$$\phi_{\hat{\varepsilon}} = (1-\tau)\frac{(1-g\hat{\varepsilon})R - cRg\hat{\varepsilon}}{1+\rho}; \quad \phi_\tau = \frac{1-\phi}{1-\tau}; \quad \phi_{\hat{\varepsilon}\tau} = \frac{-\phi_{\hat{\varepsilon}}}{1-\tau}$$

Eq. (16) for $\Delta(\cdot)$ still applies. Differentiating it with respect to τ and using these properties of $\phi(\cdot)$ yields:

$$\Delta_\tau = \frac{\phi_{\hat{\varepsilon}\tau}}{1-\phi} + \frac{\phi_{\hat{\varepsilon}}\phi_\tau}{(1-\phi)^2} = 0$$

Performing a comparative static analysis on (21) as before and using $\Delta_\tau = 0$, Lemma 3 now becomes, for $\hat{\varepsilon}^*$ in the interior:

$$\frac{d\hat{\varepsilon}^*}{d\tau} = 0$$

Thus, a cash-flow tax with monitoring costs deductible would not affect bankruptcy risk for firms with $\hat{\varepsilon}^*$ in the interior.

To find the effect on leverage, (33) becomes, for firms with $\hat{\varepsilon}^*$ in the interior:

$$\frac{d\phi}{d\tau} = \phi_\tau = \frac{1-\phi}{1-\tau} > 0 \tag{35}$$

since $d\hat{\varepsilon}^*/d\tau = 0$. Therefore, while the tax does not affect bankruptcy risk, it does increase leverage and investment so is not neutral.

Next, consider the effect of the tax on expected profits of the firm. Lemma 2 still applies, so by (14), $\overline{\pi}$ is given by:

$$\overline{\pi} = \frac{1-\tau}{1-\phi}\frac{Rg}{2}(\varepsilon_{\max} - \hat{\varepsilon})^2 \equiv D\frac{Rg}{2}(\varepsilon_{\max} - \hat{\varepsilon})^2$$

By (35), $D \equiv (1-\tau)/(1-\phi)$ is independent of τ, so expected profits are as well.[8] Thus, while the tax increases leverage and therefore K, it leaves expected after-tax profits unchanged.

Finally, tax revenue will rise with the tax rate. Using (4) expected government revenue from the cash-flow tax is

$$\overline{T} = \tau\int_{\widehat{R}}^{\overline{R}}(\overline{\varepsilon}R - \rho - \delta)\frac{E}{1-\phi(\cdot)}dH(R) \equiv \tau\overline{B} \tag{36}$$

[8] Proof:

$$D_\tau = -\frac{1}{1-\phi} + \frac{1-\tau}{(1-\phi)^2}\phi_\tau = -\frac{1}{1-\phi} + \frac{1-\tau}{(1-\phi)^2}\frac{1-\phi}{1-\tau} = 0 \qquad \blacksquare$$

where $H(R)$ is the distribution of entrepreneur types and $\overline{B}$ is the expected tax base. Given that $\widehat{R}$ is not affected by the tax, differentiating $\overline{T}$ yields:

$$\frac{d\overline{T}}{d\tau} = \overline{B} + \tau \int_{\widehat{R}}^{\overline{R}} (\bar{\varepsilon}R - \rho - \delta) \frac{E}{(1-\phi)^2} \frac{d\phi}{d\tau} dH(R) \ > 0 \tag{37}$$

where the inequality follows from (35).

Consider now the effect of the tax on constrained social surplus S. Using $\overline{\Pi} - (1+\rho)E = (1-\tau)S$, we have:

$$S = \frac{\overline{\Pi} - (1+\rho)E}{1-\tau}$$

Since a tax increase leaves $\overline{\Pi} = \overline{\pi}E$ unchanged, it will increase S. In effect, the tax induces the firms to increase leverage while keeping $\hat{\varepsilon}$ constant. This increases K and as can be seen from (25) S increases. Equivalently, the increase in K holding $\hat{\varepsilon}$ constant generates more pre-tax profits, or rents. The government taxes away those profits, leaving after-tax expected profits unchanged and improving constrained social surplus. Thus, while the no-tax outcome replicates the constrained social optimum, implementing a cash-flow tax improves social outcomes without changing firms' expected profits. It does so by breaking the connection between leverage and bankruptcy risk.

The optimal tax rate for a given entrepreneur type would be that which just induced the entrepreneur to choose maximum leverage. The government cannot observe R so cannot implement optimal type-specific tax rates.

The main results of the analysis of this section are summarized in the following proposition.

Proposition 1 *With risk-neutral entrepreneurs, equilibrium has the following properties:*

i. Entrepreneurs with average product R above some threshold level $\widehat{R}$ enter the risky industry and earn a rent. For those with K in the interior, leverage ϕ and bankruptcy risk $\hat{\varepsilon}^*$ are increasing with R.

ii. In the absence of taxation, the equilibrium leverage and entry decisions are socially efficient.

iii. With costly monitoring ($c > 0$), the cash-flow tax distorts leverage but does not distort entry, but distorts leverage for inframarginal entrepreneurs. The marginal entrepreneur incurs no debt and faces no bankruptcy risk. For inframarginal entrepreneurs, the effect of the cash-flow tax on leverage is ambiguous, but results in lower bankruptcy risk than in the social optimum.

iv. With costless monitoring ($c = 0$), the cash-flow tax does not distort either leverage or entry decisions. Leverage and capital are indeterminate for the marginal entrepreneur. For inframarginal entrepreneurs, leverage and capital take their maximum values and bankruptcy risk decreases with the level of the cash-flow tax.

v. If the banks could deduct monitoring costs from the income tax on bankrupt projects, leverage would increase with the tax rate, while bankruptcy risk and expected profits would be unchanged, and social surplus would increase.

6 Extensions of the Basic Model with Risk-Neutrality

In the basic model without imperfect information, we have shown that cash-flow taxation with full loss-offset is neutral: it affects neither the extensive-margin (entry) nor the intensive-margin (leverage) decision. By the same token, the CAA and RRT systems will also be neutral as long as the cost of finance used to carry forward unused capital cost deductions or untaxed cash flows is the risk-free interest rate ρ. Neutrality no longer applies if banks cannot observe firms' outcomes and must incur a cost to verify bankruptcy, unless monitoring costs are tax-deductible. We now briefly consider three extensions to the basic model. In each case, the results are straightforward extensions to the basic case, so full analyses are omitted.

6.1 The ACE tax

The ACE is sometimes thought to be equivalent to a cash-flow tax, and constitutes a relatively simple reform of ordinary corporate tax systems. As the name implies, the ACE allows firms to deduct a cost of equity finance as well as interest on debt. In the context of our basic model, instead of deducting ρK as the cost of finance, the firm deducts $\rho E + rB$. Tax liabilities in (4) become $\widetilde{T} = \tau\big(\tilde{\varepsilon}RK - \rho E - rB - \delta K\big)$. Since $r > \rho$ as long as $\hat{\varepsilon} > 0$, the interest deduction encourages borrowing by the firm, so the intensive margin decision is distorted if leverage is in the interior.

However, the extensive margin will remain undistorted under the ACE tax. With the tax liabilities given above, ex post after-tax profits become

$$\widetilde{\Pi} = (1-\tau)\Big(\tilde{\varepsilon}RK + (1-\delta)K - rB\Big) - B + \tau K + \tau\rho E \tag{38}$$

and the zero-expected profits condition for banks is

$$(1+\rho)B = (1+r)B(1-g\hat{\varepsilon}) + (1-\tau)RKg\frac{\hat{\varepsilon}^2}{2} + \Big((1-\tau)(1-\delta)K + \tau K + \tau\rho E\Big)g\hat{\varepsilon} - cRKg\frac{\hat{\varepsilon}^2}{2} \tag{39}$$

Using (38), (39), as well as (1) and (2), the expected profits of the entrepreneur, $\overline{\Pi} \equiv \int_{\hat{\varepsilon}}^{\varepsilon_{\max}} \widetilde{\Pi} g d\tilde{\varepsilon}$, can be written as

$$\overline{\Pi} = \left(\frac{1}{1-\phi(\cdot)} \Big((1-\tau)(\overline{\varepsilon}R - \delta) + \tau r(1-g\hat{\varepsilon}) - \rho - \frac{cRg\hat{\varepsilon}^2}{2} \Big) + \tau\rho - \tau r(1-g\hat{\varepsilon}) + 1 + \rho \right) E$$

$$\equiv \overline{\pi}(\hat{\varepsilon}, R, \tau, c)E \tag{40}$$

Entry decisions satisfy $\overline{\pi}(\hat{\varepsilon}, R, \tau, c)E \geqslant (1+\rho)E$, and the cutoff value $\widehat{R}$ is such that

$$\frac{1}{1-\phi(\cdot)} \Big((1-\tau)(\overline{\varepsilon}\widehat{R} - \delta) + \tau r(1-g\hat{\varepsilon}) - \rho - \frac{c\widehat{R}g\hat{\varepsilon}^2}{2} \Big) + \tau\rho - \tau r(1-g\hat{\varepsilon}) = 0 \tag{41}$$

As shown earlier, $\hat{\varepsilon} = 0$ for the marginal entrepreneur when there are monitoring costs. Therefore, $r = \rho$ for the marginal entrepreneur and (41) reduces to (32), so $\widehat{R}$ is independent of the tax.

6.2 Earned income in the risk-free sector

Suppose next that entrepreneurs who choose not to enter the risky sector obtain a fixed income denoted y in the risk-free sector, and get to invest their wealth E at the risk-free interest rate as well. In this case, an entrepreneur will enter as long as $\overline{\Pi} = \overline{\pi}(\hat{\varepsilon}, R, \tau, c)E \geqslant (1+\rho)E + y$, by analogy to (23). Consider Case I where $c = 0$. Eq. (24) determining $\widehat{R}$ becomes

$$(1-\tau)(\overline{\varepsilon}\widehat{R} - \delta - \rho) = \frac{y}{E}$$

Thus, entry would appear to be deterred, though the intensive-margin decision would not be affected.

Neutrality would be achieved if income in the risk-free sector, y, were taxed at the same tax rate applied to entrepreneurs' cash-flows, τ. Kanniainen and Panteghini (2012) also find that cash-flow taxation does not affect the choice of entrepreneurs to undertake a risky investment as long as the same tax rate applies to earnings in the safe occupation.

One could complicate matters further by assuming that part of the income earned in the risky sector reflects labor income of the entrepreneur. If that income were y and the entrepreneur was allowed to deduct it from cash flows, neutrality would prevail as long as the income y earned in the risk-free sector was taxed at the same rate as income y attributed to the entrepreneur in the risky sector.

6.3 Minimum capital requirements

In the basic model, we assumed that the entrepreneurs invested all their asset wealth E in the risky firm, and could borrow on top of that. Marginal entrepreneurs would either choose $E = K$ in the case where $c > 0$, or were indifferent about the amount of borrowing with $c = 0$. This accounted for the fact that marginal entrepreneurs might as well choose $\hat{\varepsilon} = 0$ in which case $r = \rho$, resulting in no distortion of the entry decision.

Suppose however that there is a minimum level of $K > E$ that is needed for a firm to operate. This implies that all firms must borrow at least some fixed amount, and generally there will be some bankruptcy risk, $\hat{\varepsilon} > 0$ (although that is not necessarily the case). Consider Cases I and II above. In Case I where $c = 0$, the cash-flow tax will still be neutral. The marginal entrepreneur will still be indifferent to leverage so does not mind taking on the minimum required B, and the inframarginal firms will all go to the maximum K as before.

In case II with $c > 0$, in the absence of the minimum K, the marginal firm chooses $\hat{\varepsilon} = 0$, so there is no risk of bankruptcy and $r = \rho$. Inframarginal firms have an incentive to borrow as in Case I, but they do not go to the maximum K because of the additional monitoring cost c faced by the banks, which serves to increase r. When the minimum capital constraint is imposed, marginal firms with productivity $\widehat{R}$ now have to go to the minimum, and if that causes $\hat{\varepsilon}$ to become positive for them, their entry and leverage decisions will both be distorted by the cash-flow tax.

The main findings of this section are as follows.

Proposition 2 *Under risk-neutrality, the following holds:*

i. The equilibrium under the ACE tax is inefficient. The tax encourages excessive borrowing for firms with leverage in the interior, but leaves entry decisions undistorted.

ii. If potential entrepreneurs who do not enter the risky sector earn an alternative income in the risk-free sector, in addition to the risk-free return on their wealth, the cash-flow tax will be neutral with respect to both entry and leverage as long as income earned in the risk-free sector is taxed at the same rate as cash flows in the risky sector. Otherwise, entry decisions will be distorted.

iii. If there is a minimum capital requirement greater than E, the cash-flow tax will remain neutral if monitoring is costless. With costly monitoring, if the minimum capital requirement leads the marginal entrepreneur to face strictly positive bankruptcy risk, entry and leverage will be distorted.

7 Risk-Averse Entrepreneurs

Recall that all consumption takes place at the end of the period. Entrepreneurs that enter the risky industry invest all their wealth in their firm at the beginning of the period and consume the after-tax profits $\widetilde{\Pi}$ at the end. Let their end-of-period expected utility V be:

$$V = \frac{1}{1-\gamma}\int_{\hat{\varepsilon}}^{\varepsilon_{\max}} \widetilde{\Pi}^{1-\gamma} g d\tilde{\varepsilon} \tag{42}$$

where $0 < \gamma < 1$, so the entrepreneur's utility function exhibits constant relative risk aversion. Using (7) and (2), expected utility becomes:

$$V = \frac{\big((1-\tau)RK\big)^{1-\gamma}}{1-\gamma}\int_{\hat{\varepsilon}}^{\varepsilon_{\max}} (\tilde{\varepsilon}-\hat{\varepsilon})^{1-\gamma} g d\tilde{\varepsilon} = \frac{E^{1-\gamma}}{1-\gamma}\Big(\frac{R(1-\tau)}{1-\phi(\cdot)}\Big)^{1-\gamma}(\varepsilon_{\max}-\hat{\varepsilon})^{2-\gamma}\frac{g}{2-\gamma} \tag{43}$$

Alternatively, using (14) for $\overline{\pi}(\hat{\varepsilon}, R, \tau, c)$, expected utility may be written:[9]

$$V = \frac{E^{1-\gamma}}{1-\gamma}\overline{\pi}(\hat{\varepsilon}, R, \tau, c)^{1-\gamma}(\varepsilon_{\max}-\hat{\varepsilon})^{\gamma}\frac{2^{1-\gamma}g^{\gamma}}{2-\gamma} \tag{44}$$

Eqs. (43) and (44) are alternative representations of entrepreneurs' expected utility that will be useful in what follows.

As before, the entrepreneur decides whether to enter, and if so, how much to borrow from the bank, and therefore how much risk to take on. Consider these decisions in reverse order.

7.1 Intensive-margin decision

As above, the choice of leverage ϕ is equivalent to the choice of bankruptcy risk $\hat{\varepsilon}$ through (10). Differentiating (43) with respect to $\hat{\varepsilon}$ and using the definition of $\Delta(\cdot)$ in (16), we obtain after straightforward simplification:

$$\frac{dV}{d\hat{\varepsilon}} \propto \Delta(\hat{\varepsilon}, R, \tau, c) - \frac{2-\gamma}{1-\gamma}\frac{1}{\varepsilon_{\max}-\hat{\varepsilon}} \tag{45}$$

Alternatively, differentiating (44) yields:

$$\frac{dV}{d\hat{\varepsilon}} \propto \overline{\pi}_{\hat{\varepsilon}} - \frac{\gamma}{1-\gamma}\frac{\overline{\pi}}{\varepsilon_{\max}-\hat{\varepsilon}} = \overline{\pi}_{\hat{\varepsilon}} - \frac{\gamma}{1-\gamma}\frac{Rg}{2}\frac{1-\tau}{1-\phi}(\varepsilon_{\max}-\hat{\varepsilon}) \tag{46}$$

[9] Proof: Rewrite (43) as:

$$V = \frac{E^{1-\gamma}}{1-\gamma}\Big(\frac{1-\tau}{1-\phi}R\frac{g}{2}(\varepsilon_{\max}-\hat{\varepsilon})^2\Big)^{1-\gamma}\cdot\Big(\frac{g}{2}\Big)^{\gamma-1}(\varepsilon_{\max}-\hat{\varepsilon})^{2(\gamma-1)}\frac{g}{2-\gamma}(\varepsilon_{\max}-\hat{\varepsilon})^{2-\gamma}$$

Using (14), this becomes (44). ■

using the expression for $\overline{\pi}$ in (14).

Let $\hat{\varepsilon}^*$ be the optimal choice of $\hat{\varepsilon}$. If $\hat{\varepsilon}^*$ is in the interior, $dV/d\hat{\varepsilon} = 0$, and (45) and (46) give:

$$\Delta(\hat{\varepsilon}^*, R, \tau, c) = \frac{2-\gamma}{1-\gamma}\frac{1}{\varepsilon_{\max} - \hat{\varepsilon}^*} > 0 \tag{47}$$

and

$$\overline{\pi}_{\hat{\varepsilon}} = \frac{\gamma}{1-\gamma}\frac{Rg}{2}\frac{1-\tau}{1-\phi}(\varepsilon_{\max} - \hat{\varepsilon}^*) > 0 \tag{48}$$

From (47) we obtain the following analogue to Lemma 3.[10]

Lemma 6 *If $\hat{\varepsilon}^*$ is in the interior, then*

$$\frac{d\hat{\varepsilon}^*}{d\tau} \lesseqgtr 0 \quad \text{as} \quad c \gtreqless 0 \qquad \text{and} \qquad \frac{d\hat{\varepsilon}^*}{d\gamma} < 0$$

Note that Lemma 4 applies here as well, so bankruptcy risk and therefore leverage are increasing in entrepreneurial productivity: $d\hat{\varepsilon}^*/dR > 0$.

7.2 Extensive-margin decision

The entry decision involves comparing expected utility as an entrepreneur with that obtained from the safe alternative, whose end-of-period consumption is $(1+\rho)E$. Entrepreneurs will enter if $V \geqslant \big((1+\rho)E\big)^{1-\gamma}/(1-\gamma)$, or using (44),

$$\overline{\pi}(\hat{\varepsilon}^*, R, \tau, c)^{1-\gamma}(\varepsilon_{\max} - \hat{\varepsilon}^*)^{\gamma}\frac{2^{1-\gamma}g^{\gamma}}{2-\gamma} \geqslant (1+\rho)^{1-\gamma} \tag{49}$$

From (43), V is increasing in R, so the cutoff value $\widehat{R}$ will be uniquely determined. Given that $\hat{\varepsilon}$ is being optimized, $\widehat{R}$ satisfies the following lemma.[11]

[10] Proof: Differentiate the first-order condition (47) to obtain:

$$\Delta_{\tau} d\tau + \Big(\Delta_{\hat{\varepsilon}} - \frac{2-\gamma}{1-\gamma}\frac{1}{(\varepsilon_{\max} - \hat{\varepsilon}^*)^2}\Big)d\hat{\varepsilon} = 0$$

The coefficient of $d\hat{\varepsilon}$ is negative by the second-order condition. Since $\Delta > 0$ by (47), $\phi_{\hat{\varepsilon}} > 0$ by the definition of Δ in (16). Lemma 6 follows since $\Delta_{\tau} \lesseqgtr 0$ as $c \gtreqless 0$ by (17) when $\phi_{\hat{\varepsilon}} > 0$.

[11] Proof: By (49), we have for the marginal entrepreneur, using $g = 1/\varepsilon_{\max}$,

$$\overline{\pi}(\hat{\varepsilon}^*, \widehat{R}, \tau, c) = (1+\rho)\Big(\frac{1}{\varepsilon_{\max} - \hat{\varepsilon}^*}\Big)^{\gamma/(1-\gamma)}\frac{1}{2}\Big(\frac{2-\gamma}{g^{\gamma}}\Big)^{1/(1-\gamma)} = \frac{(1+\rho)}{2}(2-\gamma)^{1/(1-\gamma)}\Big(\frac{\varepsilon_{\max}}{\varepsilon_{\max} - \hat{\varepsilon}^*}\Big)^{\gamma/(1-\gamma)}$$

Using (13), (50) follows. ■

Lemma 7 *The cutoff value of R is determined by:*

$$\frac{1}{1-\phi}\Big((1-\tau)(\widehat{R}\overline{\varepsilon}-\delta-\rho)-\frac{cg\widehat{R}\hat{\varepsilon}^2}{2}\Big)=(1+\rho)\left(\frac{(2-\gamma)^{1/(1-\gamma)}}{2}\Big(\frac{\varepsilon_{\max}}{\varepsilon_{\max}-\hat{\varepsilon}^*}\Big)^{\gamma/(1-\gamma)}-1\right) \quad (50)$$

Entrepreneurs will enter the risky sector only if $R \geqslant \widehat{R}$. Those with $R < \widehat{R}$ will invest their wealth in a safe asset.

Before turning to the effect of cash-flow taxation on leverage and entry, it is useful to show that in equilibrium $\hat{\varepsilon}^* = 0$ for the marginal entrepreneur, $\widehat{R}$. Note first that the following lemma indicates when $\hat{\varepsilon}^* = 0$.[12]

Lemma 8

$$\hat{\varepsilon}^* = 0 \qquad \textit{if} \qquad \frac{\overline{\varepsilon}R}{\delta+\rho} < \frac{1}{2}\frac{2-\gamma}{1-\gamma} \quad (51)$$

Next, we can derive the conditions under which $\hat{\varepsilon}^* = 0$ for the marginal entrepreneur $\widehat{R}$. Let R_0 be the value of R such that $\hat{\varepsilon}^*$ just becomes zero. From Lemma 8, R_0 satisfies:

$$\frac{\overline{\varepsilon}R_0}{\delta+\rho} = \frac{1}{2}\frac{2-\gamma}{1-\gamma} \quad (52)$$

The expected utility of this entrepreneur becomes from (44) and using $\varepsilon_{\max} = 1/g$:

$$V_0 = \frac{E^{1-\gamma}}{1-\gamma}\overline{\pi}(0, R_0, \tau, c)^{1-\gamma}\frac{2^{1-\gamma}}{2-\gamma}$$

Using (14) for $\overline{\pi}$, (10) for ϕ, and (52), this may be written after some manipulation:

$$V_0 = \frac{(1+\rho)^{1-\gamma}E^{1-\gamma}}{1-\gamma}\frac{1}{(1-\gamma)^{1-\gamma}(2-\gamma)^{\gamma}} \quad (53)$$

Recall that for the marginal entrepreneur, $\widehat{R}$,

$$V(\widehat{R}) = \frac{(1+\rho)^{1-\gamma}E^{1-\gamma}}{1-\gamma}$$

We have that $\widehat{R} \leqslant R_0$ if $V(\widehat{R}) \leqslant V_0$, or from (53) if $(1-\gamma)^{1-\gamma}(2-\gamma)^{\gamma} \leqslant 1$, or equivalently $\gamma \geqslant 0$, where the equality applies for $\gamma = 0$. We summarize this in the following lemma.

[12] Proof: $\hat{\varepsilon}^* = 0$ if $dV/d\hat{\varepsilon}|_{\hat{\varepsilon}=0} < 0$, or $\Delta(0, R, \tau, c) < (2-\gamma)/((1-\gamma)2\overline{\varepsilon})$ by (45). By (16),

$$\Delta(0, R, \tau, c) = \frac{\phi_{\hat{\varepsilon}}(0, R, \tau, c)}{1-\phi(0, R, \tau, c)} = \frac{(1-\tau)R/(1-\rho)}{1-(1-\tau)(1-\delta)/(1-\rho)-\tau} = \frac{R}{\rho+\delta}$$

using (10) and (11). ■

Lemma 9 $\widehat{R} \lessgtr R_0$ *as* $\gamma \gtrless 0$

The implication of Lemma 9 is that $\hat{\varepsilon}^* = 0$ at $R = \widehat{R}$ when $\gamma \gtrless 0$, and this holds independent of the values of c or τ. Note that this confirms the result that we found earlier in the risk-neutral model with $\gamma = 0$ where the marginal entrepreneur choose $\hat{\varepsilon}^* = 0$ as well. In the risk-neutral case $\widehat{R} = R_0$, so $\hat{\varepsilon}^* > 0$ for all inframarginal entrepreneurs.

In the case where entrepreneurs are risk-averse so $\gamma > 0$, Lemma 9 implies that $\hat{\varepsilon}^* = 0$ for all entrepreneurs with $R \in [\widehat{R}, R_0]$. In that range, there is no risk of bankruptcy for the marginal entrepreneur, so $r = \rho$ by the banks' zero-profit conditions (9) and ex post after-tax profits in (5) can be written, using $B = K - E = E/(1-\phi) - E$:

$$\widetilde{\Pi} = \Big(\frac{1-\tau}{1-\phi}\big(\tilde{\varepsilon}R - \delta - \rho\big) + 1 + \rho\Big)E = \big(D(\tilde{\varepsilon}R - \delta - \rho) + 1 + \rho\big)E$$

where, recall, $D \equiv (1-\tau)/(1-\phi)$. Therefore, expected utility can be written:

$$V = \int_0^{\varepsilon_{\max}} \frac{\widetilde{\Pi}^{1-\gamma}}{1-\gamma} g d\tilde{\varepsilon} = \frac{E^{1-\gamma}}{1-\gamma}\int_0^{\varepsilon_{\max}} \big(D(\tilde{\varepsilon}R - \delta - \rho) + 1 + \rho\big)^{1-\gamma} g d\tilde{\varepsilon} \tag{54}$$

7.3 The effects of cash-flow taxation

We next turn to the effects of the cash-flow tax on the entrepreneurs' intensive- and extensive-margin decisions when entrepreneurs are risk-averse. As in the risk-neutral case, we begin with the general case where there is asymmetric information. We then turn to the two special cases where $c = 0$ and where monitoring costs are tax-deductible.

Suppose then that there is both risk aversion ($\gamma > 0$) and asymmetric information ($c > 0$). Lemma 9 applies here, so $\widehat{R} < R_0$ implying that $\hat{\varepsilon}^* = 0$ for the marginal entrepreneur. Consider the extensive-margin decision. The value of $\widehat{R}$ is determined where $V = (1+\rho)^{1-\gamma}E^{1-\gamma}/(1-\gamma)$ or using (54),

$$\frac{E^{1-\gamma}}{1-\gamma}\int_0^{\varepsilon_{\max}} \big(D(\tilde{\varepsilon}\widehat{R} + 1 - \delta - \rho) + \rho\big)^{1-\gamma} g d\tilde{\varepsilon} = \frac{(1+\rho)^{1-\gamma}E^{1-\gamma}}{1-\gamma} \tag{55}$$

When $\hat{\varepsilon} = 0$, we can show that D is independent of the tax rate. To see this, note that (33) simplifies to $d\phi/d\tau = \phi_\tau$. By (11), when $\hat{\varepsilon} = 0$ we have:

$$\phi_\tau = \frac{1-\phi}{1-\tau} = \frac{1}{D} > 0 \tag{56}$$

Differentiating D with respect to τ and using (56), we obtain

$$D_\tau = -\frac{1}{1-\phi} + \frac{1-\tau}{(1-\phi)^2}\phi_\tau = 0 \tag{57}$$

Thus, $\widehat{R}$ is independent of τ since D^* is. In effect, the fact that $c > 0$ is irrelevant at the extensive margin since $\hat{\varepsilon} = 0$ implies that there is no possibility of bankruptcy and therefore no need for monitoring entrepreneurs of productivity $\widehat{R}$.

The intensive margin decision is as in the base case with risk-neutrality above. Eq. (33) again applies, and since $d\hat{\varepsilon}^*/d\tau < 0$ by Lemma 6 and the sign of ϕ_τ is ambiguous by (11), $d\phi^*/d\tau \gtreqless 0$. The cash-flow tax therefore distorts the leverage decision.

7.3.1 Costless monitoring

In this case, since $c = 0$ so imperfect information is not in play, any distorting effects of taxation are due to risk-aversion. Consider first the leverage decision and focus on inframarginal entrepreneurs for whom $R > R_0$ so $\hat{\varepsilon}^* > 0$. As above, (33) applies. From Lemma 6, $d\hat{\varepsilon}^*/d\tau = 0$ when $c = 0$. Therefore, (33) simplifies to $d\phi/d\tau = \phi_\tau$. By (11), (56) applies when $c = 0$, so τ encourages leverage. However, the increase in leverage does not translate into an increase in bankruptcy risk $\hat{\varepsilon}$ since $d\hat{\varepsilon}^*/d\tau = 0$. This is analogous to the famous Domar and Musgrave (1944) result: a tax on capital income with full loss-offset encourages risk-taking by risk-averse individuals because the government is sharing the risk with the entrepreneur on actuarially fair terms. In the case of a cash-flow tax, the government is sharing the risk of the entrepreneur, and as a consequence neither private risk nor expected after-tax profits change as a result of the tax.

To verify that an increase in τ has no effect on expected profits, consider the expression for the expected rate of return on equity, $\overline{\pi}$, in (14). Since $\hat{\varepsilon}^*$ does not change with τ, $\overline{\pi}$ will also be invariant with τ if $D \equiv (1-\tau)/(1-\phi)$ is independent of τ. Differentiating D with respect to τ and using (56), we again obtain (57) when $c = 0$. Therefore, entrepreneurs are able to offset the effect of the tax on their expected profits by increasing their leverage. The increase in leverage will correspond to an increase in borrowing and investment, which in turn will increase expected before-tax profits and therefore expected tax revenue to the government since after-tax profits are unchanged. To see that expected tax revenues will rise, note that (36) still applies, and the change in expected tax revenues will again be given by (37) since $d\phi/d\tau > 0$. Finally, since both bankruptcy risk $\hat{\varepsilon}^*$ and expected profits $\overline{\pi}$ are unaffected by the tax, (44) indicates that expected utility V is also unaffected as long as leverage ϕ and K are in the interior.

Consider next those entrepreneurs, including the marginal ones, in the range $R \in [\widehat{R}, R_0]$. For them $\hat{\varepsilon}^* = 0$, but they may have positive leverage. Their expected utility V is given by (54) above. Recalling that $D \equiv (1-\tau)/(1-\phi)$, the first-order condition $\partial V/\partial\phi = 0$

reduces to:

$$\int_0^{\varepsilon_{\max}} \big(D^*(\tilde{\varepsilon}R + 1 - \delta - \rho) + \rho\big)^{-\gamma} g d\tilde{\varepsilon} = 0$$

where D^* is the optimal choice of D. This expression yields a unique value of D^* which is invariant to changes in τ, so we obtain

$$\phi = 1 - \frac{1-\tau}{D^*}, \quad \text{with} \quad \frac{d\phi}{d\tau} = \frac{1}{D^*} > 0$$

Since D^* is independent of τ, so is expected utility in (54). This is analogous to the effect of taxes on those entrepreneurs for whom $\hat{\varepsilon}^* > 0$ above. Expected utility of entrepreneurs is not affected by a tax increase, while government revenue rises. We return to the implications for social welfare below.

Finally, consider the extensive-margin decision. The value of $\widehat{R}$ is determined where $V = (1+\rho)^{1-\gamma}E^{1-\gamma}/(1-\gamma)$ or using (54),

$$\frac{E^{1-\gamma}}{1-\gamma}\int_0^{\varepsilon_{\max}} \big(D(\tilde{\varepsilon}\widehat{R} + 1 - \delta - \rho) + \rho\big)^{1-\gamma} g d\tilde{\varepsilon} = \frac{(1+\rho)^{1-\gamma}E^{1-\gamma}}{1-\gamma} \tag{58}$$

Thus, $\widehat{R}$ is independent of τ since D^* is.

In summary, if $c = 0$ and $\gamma > 0$, leverage is encouraged by the cash-flow tax, but entrepreneurs' expected profits are not affected. As well, entry is not affected. Expected tax revenues of the government increase, reflecting the fact that the increase in leverage increases before-tax profits or rents while leaving after-tax profits unchanged. Social welfare will increase if the increase in expected tax revenues is valuable to the government, and that depends on how the government evaluates the increase in risk that might accompany the tax revenues.

Consider two extreme cases. In the first, assume that ε is an idiosyncratic shock and that the government can fully pool risk. Then, expected tax revenues in (36) are non-stochastic and the increase in tax revenues (37) are unambiguously beneficial. The increase in the tax rate therefore improves social welfare.

In the other extreme, suppose ε is a common shock to all firms, so government revenue is stochastic. Total tax revenue can then be written $T(\varepsilon)$, and expected tax revenue is $\overline{T} = \mathbb{E}_\varepsilon[T(\varepsilon)]$, and continues to be given by (36). The variance of government revenue can be written:

$$\text{Var}[T] = \tau^2\Big(\int_{\widehat{R}}^{\overline{R}} \frac{R}{1-\phi} dH(R)E\Big)^2 \sigma_\varepsilon^2$$

Suppose the social value of government revenue is given by the concave function $G(T)$. Then, a second-order Taylor expansion gives:

$$\mathbb{E}_\varepsilon[G(T(\varepsilon))] \approx G(\overline{T}) + G'(\overline{T})(T(\varepsilon) - \overline{T}) + \frac{1}{2}G''(\overline{T})(T(\varepsilon) - \overline{T})^2$$

$$= G(\overline{T}) + \frac{1}{2}G''(\overline{T})\text{Var}[T]$$

If $G''(\overline{T}) < 0$, this expression is generally ambiguous in sign. The more concave is $G(T)$, the less willing is the government to take on risk, and the more likely will an increase in the tax rate reduce social welfare even though it increases expected tax revenues. If an increase in the tax rate increases social welfare, the optimal cash-flow tax rate on entrepreneur R will be the one that just causes the entrepreneur to go to the maximal leverage. Of course, since the government cannot identify entrepreneurs' types, it cannot apply the optimal type-specific tax rate.

This compares with the standard analysis of capital income taxation and risk-taking discussed by Atkinson and Stiglitz (1980) and Buchholz and Konrad (2014). If the government is better able to pool risk than individuals, the no-tax equilibrium will entail too little risk-taking. A tax on capital income that results in more risk-taking—that is, more leverage—will improve social outcomes without affecting the expected utility of individuals. In the extreme case where the government can pool risk perfectly, the increase in pre-tax expected income does not affect the total risk faced by the economy, since the risk of individuals has not changed. However, if the government is unable to pool risk any better then the private sector, private sector outcomes will be optimal. The increase in expected pre-tax income resulting from capital income taxation will be accompanied by an increase in risk that individuals were not willing to bear on their own, so excessive risk-taking will be induced from a social point of view.

7.3.2 Monitoring costs deductible

If the tax system provides a deduction to banks for the monitoring costs they incur on firms that go bankrupt, we can readily show that the cash-flow tax will not affect bankruptcy risk but will increase leverage, just as in the case with risk-neutral entrepreneurs. To see this, simply note that as shown in Section 6.1, $\Delta_\tau = 0$ when monitoring costs are deductible. Differentiating (47), and using $\Delta_\tau = 0$, Lemma 6 then becomes:

$$\frac{d\hat{\varepsilon}^*}{d\tau} = 0$$

With $d\hat{\varepsilon}^*/d\tau = 0$, the effect of the tax on leverage is given by (35). Therefore, as in the risk-neutral case with $c > 0$, when monitoring costs are deductible, the tax unambiguously increases leverage although it leaves bankruptcy risk unchanged. As in the case with costless monitoring, both expected profits $\overline{\pi}$ and expected utility V are unchanged by the tax, and constrained social surplus increases as long as the firm remains in the interior. As mentioned, allowing monitoring costs to be deductible by the bank on bankrupt projects may be difficult to implement.

7.4 A two-period interpretation

The effect of cash-flow taxation on the extensive margin depends on what we assume to be the alternative to undertaking the risky investment. In the basic model, we assume that entrepreneurs who do not enter invest their endowment E in a risk-free asset instead of in the risky firm, and consume the proceeds at the end of the period. Suppose instead that we adopt a two-period perspective. In the first period, the entrepreneur either invests E in the risky firm that generates a return in period 2, or divides E into first-period consumption and invests the rest in a risk-free asset to finance second-period consumption. This is a stark way to impose a liquidity constraint on the entrepreneur who decides to enter the risky sector, but it serves to illustrate the consequences of this liquidity constraint in the simplest way.

This model differs from the above ones only in the nature of the participation decision: the intensive-margin analysis remains the same. Let us therefore focus on the extensive-margin decision. Entrepreneurs will choose to invest in the risky asset if their expected utility from doing so, V, exceeds the utility they can obtain by consuming their wealth over the two periods and taking advantage of risk-free assets for saving. Utility from not participating is obtained by maximizing $C_1^{1-\gamma}/(1-\gamma)+\beta C_2^{1-\gamma}/(1-\gamma)$ subject to the budget $C_1 + C_2/(1+\rho) = E$. To simplify, let $\beta = 1/(1+\rho)$, so the economy is in a steady state and $C_1 = C_2$. Then, the solution is $C_1 = C_2 = (1+\rho)E/(2+\rho)$, leading to lifetime utility:

$$U_0 = \frac{E^{1-\gamma}}{1-\gamma}\Big(\frac{2+\rho}{1+\rho}\Big)^{\gamma} \tag{59}$$

Recall that V is expected utility evaluated in period 2. The entrepreneur will enter if:

$$V \geqslant (1+\rho)U_0 = \frac{(1+\rho)^{1-\gamma}E^{1-\gamma}}{1-\gamma}(2+\rho)^{\gamma}$$

For the marginal entrepreneur, we have

$$V(\widehat{R}) = \frac{(1+\rho)^{1-\gamma}E^{1-\gamma}}{1-\gamma}(2+\rho)^{\gamma} \tag{60}$$

The utility for the entrepreneur for whom $\hat{\varepsilon}^*$ was just equal to zero is again given by V_0 in (53). As before, we have that $\widehat{R} \gtreqless R_0$ as $V(\widehat{R}) \gtreqless V_0$, leading to the following lemma.

Lemma 10 *$\widehat{R} \gtreqless R_0$ as* $(2+\rho)^{\gamma}(1-\gamma)^{1-\gamma}(2-\gamma)^{\gamma} \gtreqless 1$

If $\gamma = 0$, so entrepreneurs are risk-neutral, we obtain $\widehat{R} = R_0$, so $\hat{\varepsilon}^*$ is just zero for the marginal entrepreneur. If $\gamma > 0$, $\widehat{R}$ can be greater or less than R_0. If $\widehat{R} < R_0$, we have $\hat{\varepsilon}^* = 0$ for the marginal entrepreneur. In this case, the analysis of the extensive-margin decision for Case IV above applies: entry is unaffected by τ.

Suppose however that $\widehat{R} > R_0$. This will occur if the interest rate ρ is large enough and γ is small enough. To determine the effect of τ on the entry decision, rewrite (43) as

$$V = \frac{(RE)^{1-\gamma}}{1-\gamma} D^{1-\gamma} (\varepsilon_{\max} - \hat{\varepsilon}*)^{2-\gamma} \frac{g}{2-\gamma}$$

Given that $c > 0$, differentiating D with respect to τ yields:[13]

$$D_\tau = -\frac{cRg\hat{\varepsilon}^2}{2\rho(1-\phi)^2} < 0$$

Assuming that $\hat{\varepsilon}$ is chosen optimally, we can apply the envelope theorem to V to obtain $V_\tau < 0$ using $D_\tau < 0$. This implies that $d\widehat{R}/d\tau > 0$. That is, the cash-flow tax discourages entry.

Our findings with risk-averse entrepreneurs are summarized as follows.

Proposition 3 *With risk-averse entrepreneurs, equilibrium has the following properties:*

i. There is a range of entrepreneurs with $R \in [\widehat{R}, R_0]$ *for whom there is no risk of bankruptcy, so* $\hat{\varepsilon}^* = 0$ *and* $r = \rho$*;*

ii. With costly monitoring $(c > 0)$*, the impact of the cash-flow tax on leverage is ambiguous, unless monitoring costs are deductible in which case leverage increases, while bankruptcy risk, expected profits and expected utility are unchanged, so constrained social surplus increases;*

iii. With costless monitoring $(c = 0)$*, the cash-flow tax increases borrowing and leverage. However, it is neutral with respect to bankruptcy risk. The cash-flow tax results in some risk-sharing between the government and the entrepreneur, and as a result, after-tax profits are independent of the tax rate. The effect of the tax on expected profits is offset by increased leverage. Constrained social welfare increases with the tax as long as the government is not too risk averse;*

[13] Proof: Using (11),

$$D_\tau = -\frac{1}{1-\phi} + \frac{1-\tau}{(1-\phi)^2}\phi_\tau = -\frac{1}{1-\phi} + \frac{1-\tau}{(1-\phi)^2}\Big(\frac{1-\phi}{1-\tau} - \frac{cRg\hat{\varepsilon}^2}{2\rho(1-\tau)}\Big) = -\frac{cRg\hat{\varepsilon}^2}{2\rho(1-\phi)^2} \qquad \blacksquare$$

iv. The cash-flow tax is neutral with respect to entry decisions with either costless or costly monitoring; and

v. If potential entrepreneurs who choose not to enter the risky sector allocate their wealth to finance consumption over two periods, it is possible that $\hat{\varepsilon}^* > 0$ at $\widehat{R}$. In that case, the cash-flow tax will discourage entry.

8 Adverse Selection

So far we have assumed that the banks can observe entrepreneurs' types R, but may need to engage in costly monitoring to verify bankruptcy. We now explore the consequences of entrepreneurial types being private information following Stiglitz and Weiss (1981) and de Meza and Webb (1987). To focus on adverse selection, we assume that $c = \gamma = 0$ where cash-flow taxation is fully efficient. In this case, all inframarginal entrepreneurs choose $K = \overline{K}$ and borrow the same amount $B = \overline{K} - E$, as shown earlier. Marginal entrepreneurs are indifferent about B, and it simplifies matters if we assume that they too borrow $\overline{K} - E$. The model we adopt is a variant of de Meza and Webb.

8.1 Entry decisions

Assume K and B are constant and the same for all entrepreneurs who enter. There is no intensive-margin decision, only an extensive-margin one. Assume R is distributed uniformly over $[0, \overline{R}]$ with density $h = 1/\overline{R}$. Eqs. (1)–(7) continue to apply. From (6), $\hat{\varepsilon}$ is determined as a function of R such that:

$$\hat{\varepsilon}R = \frac{(1+r)}{1-\tau}\frac{B}{K} - \frac{\tau(1+\rho)}{1-\tau} - (1-\delta) \equiv A \tag{61}$$

so $\hat{\varepsilon} = A/R$ with $d\hat{\varepsilon}/dR = -A/R^2 < 0$, and

$$\hat{\varepsilon}_\tau = \frac{(1+r)B - (1+\rho)K}{(1-\tau)^2 RK}$$

The banks cannot observe R, so must offer the same interest rate to all firms. The banks zero-profit condition (9) with $c = 0$ aggregated over all entrepreneur-types gives:

$$\int_{\widehat{R}}^{\overline{R}} (1+\rho)BhdR = \int_{\widehat{R}}^{\overline{R}} \Big((1+r)B(1-g\hat{\varepsilon}) + (1-\tau)RKg\frac{\hat{\varepsilon}^2}{2} + \big((1-\tau)(1-\delta)+\tau(1+\rho)\big)Kg\hat{\varepsilon}\Big)hdR$$

Using (61) and rearranging, this can be written as:

$$(\rho - r)B(1 - h\widehat{R}) =$$

$$-\left((1+r)BgA-(1-\tau)gK\frac{A^2}{2}-\big((1-\tau)(1-\delta)+\tau(1+\rho)\big)gKA\right)h\log\left(\frac{\overline{R}}{\widehat{R}}\right) \quad (62)$$

The same interest rate is applied to all R, but the chances of bankruptcy ($\hat{\varepsilon}$) are decreasing in R. So, high$-R$ firms will be cross-subsidizing low$-R$ firms. The marginal entrepreneur $\widehat{R}$, which has the highest $\hat{\varepsilon}$, will be paying too low an interest rate relative to its actuarially fair value. Therefore, in the no-tax case, the equilibrium will be inefficient. There will be too much entry as in de Meza-Webb (1987).

As in Boadway and Keen (2006), we can show this by comparing the expected social profits with the net expected private profits for the marginal entrepreneur, $\widehat{R}$. These are, respectively:

$$\widehat{S}=\int_0^{\varepsilon_{\max}}\big(\tilde{\varepsilon}\widehat{R}K+(1-\delta)K-(1+\rho)K\big)gd\tilde{\varepsilon}=(\overline{\varepsilon}\widehat{R}-\delta-\rho)K \quad (63)$$

and

$$\widehat{\Pi}=\int_{\hat{\varepsilon}}^{\varepsilon_{\max}}\big(\tilde{\varepsilon}\widehat{R}K+(1-\delta)K-(1+r)B\big)gd\tilde{\varepsilon}=\widehat{R}Kg\Big(\frac{\varepsilon_{\max}^2}{2}-\frac{\hat{\varepsilon}^2}{2}\Big)+\big((1-\delta)K-(1+r)B\big)(1-g\hat{\varepsilon})$$

Subtracting $(1+\rho)E$ from both sides of the equation above, and using $\overline{\varepsilon}=\varepsilon_{max}/2=1/2g$ and $\hat{\varepsilon}=A/\widehat{R}$, we obtain

$$\widehat{\Pi}-(1+\rho)E-\widehat{S}=(\rho-r)B-\Big(Kg\frac{A^2}{2\widehat{R}}+(1-\delta)Kg\frac{A}{\widehat{R}}-(1+r)Bg\frac{A}{\widehat{R}}\Big)$$

Substituting the banks' zero-profit condition above, with $\tau=0$, and using $h-1/\overline{R}$, we can derive the following:

$$\widehat{\Pi}-(1+\rho)E-\widehat{S}=(\rho-r)B\left(1-\left[\frac{\widehat{R}/\overline{R}}{1-\widehat{R}/\overline{R}}\log\left(\frac{\overline{R}}{\widehat{R}}\right)\right]^{-1}\right)$$

Given that $\rho<r$, we can readily show that[14]

$$\widehat{\Pi}-(1+\rho)E-\widehat{S}>0 \quad (65)$$

Therefore, there is too much entry in equilibrium relative to the efficient level. The marginal entrepreneur generates negative expected social surplus but chooses to undertake the risky investment and is financed at the credit market equilibrium.

[14] This follows from noting that $0<\widehat{R}/\overline{R}<1$, which implies

$$1-\left[\frac{\widehat{R}/\overline{R}}{1-\widehat{R}/\overline{R}}\log\left(\frac{\overline{R}}{\widehat{R}}\right)\right]^{-1}<0 \quad (64)$$

8.2 The impact of taxation

Under the cash-flow tax, the expected private profit of a type–R entrepreneur, (12), still applies. Using $g = 1/\varepsilon_{max}$, $\overline{\varepsilon} = \hat{\varepsilon}_{max}/2$ and $\hat{\varepsilon} = A/R$, we can rewrite this equation for the marginal entrepreneur $\widehat{R}$ as

$$\overline{\Pi} = (1-\tau)\overline{\varepsilon}\widehat{R}K + (1-\tau)(1-\delta)K - (1+r)B + \tau(1+\rho)K$$
$$-\frac{A}{\widehat{R}}\left((1-\tau)Kg\frac{A}{2} + (1-\tau)(1-\delta)Kg - (1+r)Bg + \tau(1+\rho)Kg\right)$$

Using the banks' zero-profit condition (62) in the equation above, and rearranging, we obtain

$$\widehat{\Pi} = (1-\tau)\overline{\varepsilon}\widehat{R}K + (1-\tau)(1-\delta)K - (1+r)B + \tau(1+\rho)K - (\rho - r)B\left[\frac{\widehat{R}h}{1-\widehat{R}h}\log\left(\frac{\overline{R}}{\widehat{R}}\right)\right]^{-1}$$

Differentiating with respect to the tax rate, holding $\widehat{R}$ constant, gives

$$\left.\frac{d\widehat{\Pi}}{d\tau}\right|_{\widehat{R}} = \widehat{\Pi}_\tau - \left.\frac{dr}{d\tau}\right|_{\widehat{R}} B\left(1 - \left[\frac{\widehat{R}/\overline{R}}{1-\widehat{R}/\overline{R}}\log\left(\frac{\overline{R}}{\widehat{R}}\right)\right]^{-1}\right) \tag{66}$$

where $\widehat{\Pi}_\tau = -\left(\overline{\varepsilon}\widehat{R} - \delta - \rho\right)K$. By (32), $\overline{\varepsilon}\widehat{R} - \delta = \rho$, so $\widehat{\Pi}_\tau = 0$. The banks' zero expected profit condition (62) determines the equilibrium interest rate, given $\widehat{R}$. From (62), and using (61), we can derive the following[15]

$$\left.\frac{dr}{d\tau}\right|_{\widehat{R}} = \frac{-\left(\hat{\varepsilon}\widehat{R}/2 - \delta - \rho\right)K}{B\left(1 - \frac{\varepsilon_{max}}{\hat{\varepsilon}}\left[\frac{\widehat{R}h}{1-\widehat{R}h}\log\left(\frac{\overline{R}}{\widehat{R}}\right)\right]^{-1}\right)} < 0 \tag{67}$$

Given (64), this implies that

$$\left.\frac{d\widehat{\Pi}}{d\tau}\right|_{\widehat{R}} < 0$$

The cash-flow tax decreases the expected private profit of the marginal entrepreneur, holding $\widehat{R}$ constant. Therefore, the tax discourages entry. Given that there is too much entry in the absence of taxation, the tax tends to mitigate the inefficiency of the credit market equilibrium.

The findings of the analysis with adverse selection are as follows.

Proposition 4 *If banks cannot observe entrepreneurial types, equilibrium has the following properties:*

[15] The numerator of this expression is positive. Given (64), the denominator is negative. It follows that $dr/d\tau < 0$.

i. The same interest rate applies to all entrepreneur types and too many entrepreneurs undertake risky investments at the credit market equilibrium relative to the efficient level;

ii. The cash-flow tax reduces the equilibrium interest rate; and

iii. Taxation reduces the number of entrepreneurs who undertake risky investments relative to the equilibrium without taxation.

9 Concluding Remarks

In this paper, we analyzed the impact of cash-flow business taxation on firms' choice of leverage and on decisions to enter a risky industry in a setting where entrepreneurs may be risk-averse and face bankruptcy risk, and where there is asymmetric information between entrepreneurs and financial intermediaries. We focused in particular on whether the neutrality of cash-flow taxation continues to hold under these features. The main results of the analysis are as follows.

With risk-neutral entrepreneurs, cash-flow taxation taxes rents only. Without asymmetric information in the credit market (i.e. with costless monitoring), the cash-flow tax affects neither entry nor leverage decisions, so the standard neutrality results apply. CAA and RRT systems are also neutral in this case provided that the risk-free interest rate is used to carry forward untaxed cash flows. When banks must undertake costly monitoring of firms that declare bankruptcy, the tax does not affect entry decisions, given that the marginal entrepreneur earns no rent, but it distorts leverage. If banks can obtain a tax deduction for the monitoring costs they incur on the bankrupt firms they seize, the tax will increase leverage but will leave bankruptcy risk and the firms' after-tax profits unchanged. Surprisingly, the tax will increase social welfare in this case. By inducing firms to increase leverage, and therefore capital, the tax leads to higher pre-tax profits, or rents, without affecting bankruptcy risk. These additional pre-tax profits are effectively taxed away by the government leading to a higher constrained social surplus.

In contrast to the cash-flow tax, or the CAA and RRT equivalent, the ACE is generally not neutral with respect to entry and leverage even in the absence of monitoring costs. As long as there is some bankruptcy risk, the ACE will induce firms to undertake excessive borrowing leading to higher bankruptcy risk.

With risk-averse entrepreneurs, cash-flow taxation taxes both rents and return to risk. Without monitoring costs, or if monitoring costs are deductible, the cash-flow tax increases leverage, but is neutral with respect to bankruptcy risk, expected profits and expected utility. The cash-flow tax involves some risk-sharing between the government and firms, and increases social welfare in this case for firms will levels of capital in the interior. The tax increases rents and tax revenues without affecting expected utility. Interestingly, this holds whether or not the government can pool risk better than the private sector which contrasts with standard results from the capital income tax literature.

We then considered the case where banks cannot observe the productivity of entrepreneurs ex ante, so are faced with an adverse selection problem as in Stiglitz and Weiss (1981) and de Meza and Webb (1987), among others. In this case, if banks are unable to offer separating contracts, all entrepreneurs will face the same interest rate. In contrast to the case considered here, the market equilibrium without taxation will be inefficient along the extensive margin and cash-flow taxation will generally not be neutral. In particular, there will be excessive entry by the least-productive entrepreneurs to take advantage of the favorable interest rate. A cash-flow tax discourages entry, thereby improving efficiency. It would be interesting to extend the analysis to the case where banks are able to offer contracts in which the interest rate varies with the size of loan, so firms can be separated by type. There will be informational rents that might influence the effect of cash-flow taxes.

References

Atkinson, Anthony B. and Joseph E. Stiglitz (1980) *Lectures on Public Economics* (New York: McGraw-Hill) 97–127.

Australian Treasury (2010), *Australia's Future Tax System* (Canberra: Commonwealth of Australia) (the Henry Report).

Bernanke, Ben and Mark Gertler (1989), 'Agency Costs, Net Worth, and Business Fluctuations,' *American Economic Review* 79, 14–31.

Bernanke, Ben, Mark Gertler and Simon Gilchrist (1996), 'The Financial Accelerator and the Flight to Quality,' *The Review of Economics and Statistics* 78, 1–15.

Bernanke, Ben, Mark Gertler and Simon Gilchrist (1999), 'The Financial Accelerator in a Quantitative Business Cycle Framework,' in John B. Taylor and Michael Woodford (eds), *Handbook of Macroeconomics* (Elsevier), 1341–1393.

Boadway, Robin and Michael Keen (2015), 'Rent Taxes and Royalties in Designing Fiscal Regimes for Nonrenewable Resources,' in Robert Halvorsen and David F. Layton (eds), *Handbook on the Economics of Natural Resources* (Cheltenham, UK: Edward Elgar), 97–139.

Boadway, Robin and Neil Bruce (1984), 'A General Proposition on the Design of a Neutral Business Tax,' *Journal of Public Economics* 24, 231–9.

Boadway, Robin and Michael Keen (2006), 'Financing and Taxing New Firms under Asymmetric Information,' *FinanzArchiv* 62, 471–502.

Boadway, Robin and Motohiro Sato (2011), 'Entrepreneurship and Asymmetric Information in Input Markets,' *International Tax and Public Finance* 18, 166–92.

Boadway, Robin and Jean-François Tremblay (2005), 'Public Economics and Start-up Entrepreneurs,' in Vesa Kanniainen and Christian Keuschnigg (eds), *Venture Capital, Entrepreneurship and Public Policy* (MIT Press), 181–219.

Bond, Stephen R. and Michael P. Devereux (1995), 'On the Design of a Neutral Business Tax under Uncertainty,' *Journal of Public Economics* 58, 57–71.

Bond, Stephen R., and Michael P. Devereux (2003), 'Generalised R-based and S-based Taxes under Uncertainty,' *Journal of Public Economics* 87, 1291–1311.

Brown, E. Cary (1948), 'Business-Income Taxation and Investment Incentives,' in *Income, Employment and Public Policy: Essay in Honor of Alvin H. Hansen* (New York: Norton), 300–316.

Buchholz, Wolfgang, and Kai A. Konrad (2014), 'Taxes on Risky Returns – An Update,' Max Planck Institute for Tax Law and Public Finance, Working Paper 2014-10, Munich.

Diamond, Peter (1984), 'Financial Intermediation and Delegated Monitoring,' *Review of Economic Studies* 51, 393–414.

Domar, Evsey D. and Richard Musgrave (1944), 'Proportional Income Taxation and Risk-taking,' *Quarterly Journal of Economics* 58, 388–422.

Fane, George (1987), 'Neutral Taxation Under Uncertainty,' *Journal of Public Economics* 33, 95–105.

Garnaut, Ross and Anthony Clunies-Ross (1975), 'Uncertainty, Risk Aversion and the Taxing of Natural Resource Projects,' *Economic Journal* 85, 272–87.

Institut dEconomia de Barcelona (2013), *Tax Reform*, IEB Report 2/2013.

Institute for Fiscal Studies (1991), *Equity for Companies: A Corporation Tax for the 1990s*, Commentary 26 (London).

Kanniainen, Vesa and Paolo Panteghini (2012), 'Tax Neutrality: Illusion or Reality? The Case of Entrepreneurship,' Helsinki Center of Economic Research Discussion Paper No. 349.

Klemm, Alexander (2007), 'Allowances for Corporate Equity in Practice,' *CESifo Economic Studies* 53, 229–62.

Lund, Diderik (2014), 'State Participation and Taxation in Norwegian Petroleum: Lessons for Others?,' *Energy Strategy Reviews* 3, 49–54.

Meade, James E. (1978), *The Structure and Reform of Direct Taxation*, Report of a Committee Chaired by Professor James Meade (London: George Allen and Unwin).

De Meza, David, and David C. Webb (1987), 'Too Much Investment: A Problem of Asymmetric Information,' *Quarterly Journal of Economics* 102, 281–92.

Mintz, Jack (1981), 'Some Additional Results on Investment, Risk-Taking, and Full Loss Offset Corporate Taxation with Interest Deductibility,' *Quarterly Journal of Economics* 96, 631–42.

Mirrlees, Sir James, Stuart Adam, Timothy Besley, Richard Blundell, Stephen Bond, Robert Chote, Malcolm Gammie, Paul Johnson, Gareth Myles and James Poterba (2011), *Tax by Design: The Mirrlees Review* (London: Institute for Fiscal Studies).

de Mooij, Ruud (2011), 'Tax Biases to Debt Finance: Assessing the Problem, Finding Solutions,' IMF Staff Discussion Note, SDN/11/11, Washington.

Panteghini, Paolo, Maria Laura Parisi, and Francesca Pighetti (2012), 'Italys ACE Tax and its Effect on Firm's Leverage,' CESifo Working Paper No. 3869, Munich.

President's Advisory Panel on Federal Tax Reform (2005), *Simple, Fair and Pro-Growth: Proposals to Fix Americas Tax System* (President's Advisory Panel: Washington).

Princen, Savina (2012), 'Taxes Do Affect Corporate Financing Decisions: The Case of Belgium ACE,' CESifo Working Paper No. 3713, Munich.

Sandmo, Agnar (1979), 'A Note on the Neutrality of the Cash Flow Corporation Tax,' *Economics Letters* 4, 173–76.

Stiglitz, J.E. and A. Weiss (1981), 'Credit Rationing in Markets with Imperfect Information,' *American Economic Review* 71, 393–410.

Townsend, R. (1979), 'Optimal Contracts and Competitive Markets with Costly State Verification,' *Journal of Economic Theory* 21, 265–293.

United States Treasury (1977), *Blueprints for Basic Tax Reform* (Washington: Treasury of the United States).

Williamson, S. (1986), 'Costly Monitoring, Financial Intermediation and Equilibrium Credit Rationing,' *Journal of Monetary Economics* 18, 159–79.

Williamson, S. (1987), 'Costly Monitoring, Loan Contracts, and Equilibrium Credit Rationing,' *Quarterly Journal of Economics* 101, 135–45.

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